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SENSITIVITY AND UNCERTAINTY OF THE RADIAL MODE ANALYSIS FOR TURBOMACHINERY APPLICATIONS

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ABSTRACT

Air traffic is expected to increase in the coming decades. This leads to a significant burden on residents and on the environment close to airports due to the emitted noise. Therefore, innovative noise reduction concepts applied to airplane engines are required. In the present research, the sound propagation through a low pressure turbine (LPT) is investigated by means of the Radial Mode Analysis (RMA). This method identifies the transmitted sound field within the LPT in form of propagating acoustical modes. Before measurements are carried out, a sensitivity analysis of the parameters influencing the overall error due to the application of the RMA is performed. The results are used to define requirements for a high-quality measurement and support improvements of future setups.

NOMENCLATURE

Γ	Random amplitude	k_2	Second exponential constant
ω	Angular frequency	$k_{mn}^{\pm}$	Axial wave number
φ	Circumferential coordinate	m_t	Total azimuthal mode orders
ξ_{mn}	Eigenvalues	m	Azimuthal mode order
Ω	Rotor angular speed	M_x	Axial Mach number
$A_{mn}^{\pm}$	Radial mode amplitude	N_{φ}	Azimuthal measuring positions
$(A_m)_c$	Calculated azimuthal amplitude	n	Radial mode order
$(A_m)_s$	Simulated azimuthal amplitude	p	Sound pressure
B	Number of rotor blades	R	Outer duct radius
BPF	Blade Passing Frequency	r	Radial coordinate
c	Speed of sound	r_0	Fixed radial coordinate
f_{mn}	Radial eigenfunction	s	Integral multiple
f_{co}	Cut-On frequency	t	Time
h	Harmonic of the BPF	V	Number of stator vanes
k	Wave number	x	Axial coordinate
k_1	First exponential constant	x_0	Fixed axial coordinate

INTRODUCTION

The Radial Mode Analysis (RMA) is an evaluation technique developed to study the sound transmission mechanisms of the discrete tonal noise components present in turbomachinery (Holste and Neise 1997, Taddei et al. 2009). The RMA provides a decomposition of the sound field into a distribution of acoustical modes propagating within the turbomachine at the blade passing frequency BPF (defined as the product of the rotor blades and the angular speed of the rotating machine) and its harmonics (Holste and Neise 1997). Thus, the sound field is decomposed into radial and azimuthal modes. The latter are computed via spatial spectral analysis of the signals from microphones at several pheripheral positions. Once determinded, one may expect measurement errors in the azimuthal modes. As a way to predict the influence of possible measurement errors, Holste and Neise (1997) performed an accuracy check of the RMA technique through the addition of constant amplitude noise to previously generated azimuthal modes. The simulation showed that in spite of the assumed error, no significant amplitude change in the original distribution of azimuthal modes was noticed. The addition of random noise to a specified acoustical mode distribution does not alter the resulting amplitudes of the modes after implementation of the RMA. Enghardt (1999) pointed out, that the mode amplitudes were predicted reasonably well with Gaussian noise added to an original simulated acoustic field. A further study aiming to optimize sensor arrangements for the implementation of the RMA was conducted by Tapken and Enghardt (2006). Four different types of measurement arrangements were simulated assuming a number of available sensors and a maximum permisible innacuracy for the expected results. The simulation revealed substantial differences in the outcome of the RMA for different measurement configurations. On the one hand, favourable results were obtained for radial sensor arrangements intended to measure sound fields with high frequency modal components. On the other hand, sensors with the wall of the duct positioned flush favour the measurement of the sound field enclosed in ducts of high hub-to-tip ratio.

Despite the acknowledged importance of the previously mentioned parameters, additional data acquisition and turbine-related operating variables may influence the outcome of the RMA. Therefore, the present research focuses on determining the influence of those variables over the azimuthal mode amplitudes. Selected parameters examined in the context of this work include (1) the signal-to-noise ratio (SNR) of the employed sensors, (2) the number of triggered revolutions of the rotor, (3) the circumferential spacing of the measuring positions, and (4) the turbine operating temperature. These four parameters are subsequently restricted within realistic ranges. The influence of the selected parameters on the RMA output is achieved through a numeric simulation. This simulation determines the difference between a calculated modal amplitude derived from an empirical mathematical model and the amplitude of a generated azimuthal mode based on the solution of the wave equation for cylindrical geometries. In this way, the performed simulations provide a sensitivity analysis to define optimum ranges for the data acquisition parameters previously listed. This is done to guarantee a minimal measurement uncertainty. This error is expected to be lower than 1% for an actual measurement campaign.

THEORETICAL BACKGROUND

The acoustic field prevailing inside a turbomachine is composed of several superimposed sound pressure waves. These pressure waves are space and time dependent, resulting in a locally fluctuating sound pressure. The acoustical modes are described by the linearized wave equation in cylindrical coordinates (x, r, φ) and represented by the general expression (Ghiladi 1981, Holste and Neise 1997, Taddei et al. 2009)

$$p(x, r, \varphi, t) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{\infty} \left(A_{mn}^{+} \mathrm{e}^{ik_{mn}^{+}x} + A_{mn}^{-} \mathrm{e}^{ik_{mn}^{-}x} \right) f_{mn} \left(\xi_{mn} \frac{r}{R} \right) \mathrm{e}^{im\varphi} \mathrm{e}^{i\omega t} \tag{1}$$

where p is the sound pressure, A_{mn}^{+} and A_{mn}^{-} are the complex radial mode amplitudes. k_{mn}^{+} and k_{mn}^{-} are the axial wave numbers, ω is the angular frequency, t is the time and R is the outer radius of the considered geometry. The radial dependent term f_{mn} with associated eigenvalues ξ_{mn}, is the solution of the general mth order Bessel differential equation. The analytical solution stands for the sound field generated by an infinite series of acoustical modes (m, n). Each mode combination (m, n) represents two sound waves with common angular frequency ω and respective amplitudes A_{mn}^{+} and A_{mn}^{-}. Both waves propagate in axial direction but in opposite senses. While propagating, they rotate and displace radially as well. The ordered pair (m, n) defines an specific acoustical mode. The index m, known as circumferential or azimuthal mode index, defines the number of sound pressure cycles taking place in the circumferential direction. The index n, the radial mode order, gives an indication of the sound pressure nodes (zero pressure zones) present along the radial direction.

Propagating modal structure

Theoretically, an infinite number of acoustical modes constitute the sound field within a turbomachine (Eq.1). Physical and geometrical conditions allow only the propagation of a certain quantity of them. The axial wave number $k^{\pm}_{mn}$, defines which mode combinations are able to propagate. Assuming uniform air flow, the axial wave number is defined as

$$k^{\pm}_{mn} = \frac{k}{1-M_x^2} \cdot \left[-M_x \pm \sqrt{1-(1-M_x^2)\cdot\left(\frac{\xi_{mn}}{k\cdot R}\right)^2} \right] \tag{2}$$

where k is the wave number and M_x is the axial Mach number. The mode combination (m, n) propagates if $k^{\pm}_{mn}$ is real. In that case, the terms $A^{+}_{mn}\mathrm{e}^{ik^{+}_{mn}x}$ and $A^{-}_{mn}\mathrm{e}^{ik^{-}_{mn}x}$ in Eq.1 remain unchanged. If $k^{\pm}_{mn}$ is complex, the corresponding axial term results in an exponential decay. Finally, the case $k^{\pm}_{mn}$ = 0 indicates a limiting situation, leading to a Cut-On frequency

$$f_{co} = \frac{\xi_{mn}\cdot c}{2\pi\cdot R}\cdot\sqrt{1-M_x^2}, \tag{3}$$

at which a mode (m, n) is able to propagate. Should $2\pi f_{co}$ be greater than the angular frequency ω of a particular mode, then it will contribute to the whole acoustical field present inside the turbomachine. The specific modes propagating through such a device result from the rotor-stator interaction and are specified by a simple mathematical relation proposed by Tyler and Sofrin (1962):

$$m = hB \pm sV \tag{4}$$

h denotes the harmonic $(h = 1, 2, 3, ...)$ of the BPF, B the number of rotor blades, V the number of stator vanes and s an integer $(s = ..., -2, -1, 0, 1, 2, ...)$. It takes in account the periodic behavior of the pressure oscillations caused by a rotor blade passing a stator vane and predicts all posible excitable circumferential modes m.

Radial Mode Analysis (RMA)

The computation of the propagating sound field in a turbomachine is based on the determination of the radial-mode amplitudes $A^{\pm}_{mn}$ (Eq. 1). This requires first the measurement of sound pressure at several axial and circumferential positions by means of a microphone array. This action is followed by the application of the Radial Mode Analysis (Holste and Neise 1997), a methodology used for the modal decomposition of a general in-duct acoustic field. The RMA requires rotor-synchronized sound pressure measurements as input. The synchronization provides a common data acquisition start for each recorded signal. The data is subsequently used to reconstruct at specific times the instantaneous circumferential pressure distribution. The application of a spatial Fourier Transformation over this data set constitutes the first action performed by the RMA:

$$A_m(x, r_0) = \frac{1}{N_\varphi}\sum_{l=0}^{N_\varphi-1} p\left(x_0, r_0, \varphi_l\right)\mathrm{e}^{-im\varphi_l} \tag{5}$$

where $p\left(x_0, r_0, \varphi_l\right)$ is the measured pressure at a specific position, with N_φ representing the total number of azimuthal measuring points. This reveals the amplitude and phase of the dominant circumferential modes A_m present in the sound field at the BPF or one of its harmonics. The radial-mode amplitudes $A^{\pm}_{m,n}$ are then analytically determined by grouping together the radial and axial terms of Eq.1 in A_m:

$$A_m\left(x, r\right) = \sum_{n=0}^{\infty}\left(A^{+}_{mn}\mathrm{e}^{ik^{+}_{mn}x} + A^{-}_{mn}\mathrm{e}^{ik^{-}_{mn}x}\right) f_{mn}\left(\xi_{mn}\frac{r}{R}\right) \tag{6}$$

with A_m determined from the spatial Fourier Transformation (Eq. 5), the amplitude and phase of the radial modes are calculated by solving the system of linear equations represented by Eq.6.

SENSITIVITY ANALYSIS

The resulting amplitude of the dominating modes determined through the RMA is influenced by data acquistion and turbine-related operating parameters. In order to study specifically their impact on the amplitude of the azimuthal modes A_m, a simulation routine is developed to perform a sensitivity analysis. The generation of the modal structure behind the first turbine stage is based on the operating parameters of the machine and the excitable azimuthal modes according to Tyler and Sofrin (1962). The generated acoustic field contains propagating mode combinations (m, n) up to the second harmonic of the BPF. With this information, the axial wave numbers among other parameters are calculated for all modes present in the simulated modal structure. The simulated sound field is assumed to propagate under uniform flow conditions and takes into consideration downstream propagating radial modes only A^+_{mn}, ie., acoustic modes propagating in the positive x-direction. So far, this turns to be an almost complete representation of the analytical solution (Eqn. 1), but still lacks the radial-mode amplitudes $A^\pm_{mn}$. These amplitudes are computed based on some basic assumptions and on the following simple mathematical model:

$$A^+_{mn} = \left(A_1 \mathrm{e}^{-\frac{1}{2}\left(\frac{m}{m_t}\right)^2} \right) \cdot \left(A_2 \mathrm{e}^{-k_1 h} \right) \cdot \left(A_3 \mathrm{e}^{-k_2 n} \right) \cdot \Gamma(x) \tag{7}$$

This model considers an exponential decay of modes generated by higher harmonic orders of the BPF as well as for radial mode orders n (Taddei et al. 2009). The decay constants are k_1 and k_2, respectively. The amplitudes A_1, A_2 and A_3 are set to one for every simulation. Since negative azimuthal mode orders m also appear in the analysis, a Gaussian shape relating amplitude to mode order m and total number of azimuthal mode orders m_t is a reasonable assumption (Holste and Neise 1997). In order to obtain a more stochastic modal composition, the amplitudes are then multiplied with randomly generated numbers Γ. These random numbers follow a Gamma distribution with an arithmetical mean of 1. Accordingly, the resulting acoustical signal, which one could measure at any given point inside the duct, can now be determined with the information obtained from the mathematical model. This is done by adding all radial components belonging to each azimuthal mode with Eq.6. The result is a simulated amplitude $(A_m)_s$. With this in mind, the analytical solution (Eqn. 1) is written down in terms of the simulated azimuthal amplitude:

$$p_S(x, r, \varphi, t) = \sum_{m=-\infty}^{\infty} (A_m)_s \, \mathrm{e}^{im\varphi} \mathrm{e}^{i\omega t} \tag{8}$$

Afterwards, the analytical solution (Eq.1) - using the previously determined $(A_m)_s$ - is used to generate a set of sound pressure signals. These signals recall measurements made by a microphone array, with the flexibility of generating any quantity of spatial measuring positions. The previously generated signals are then analyzed by the RMA method, resulting on the calculated azimuthal mode amplitudes, designated by $(A_m)_c$. Finally, both amplitudes - simulated and calculated - are compared directly by determining the relative error between them:

$$Error\ A_m[\%] = \left| \frac{(A_m)_s - (A_m)_c}{(A_m)_s} \right| \tag{9}$$

The magnitude of this relative error is an indication of the influence that data acquisition and operating parameters have on an actual measurement campaign and for a specific mode combination. The chosen methodology compares the sound field that is at first expected to what would be actually measured. A schematic diagram showing the procedure for the general sensitivity analysis is presented in Fig. 1 along with the parameters being investigated.

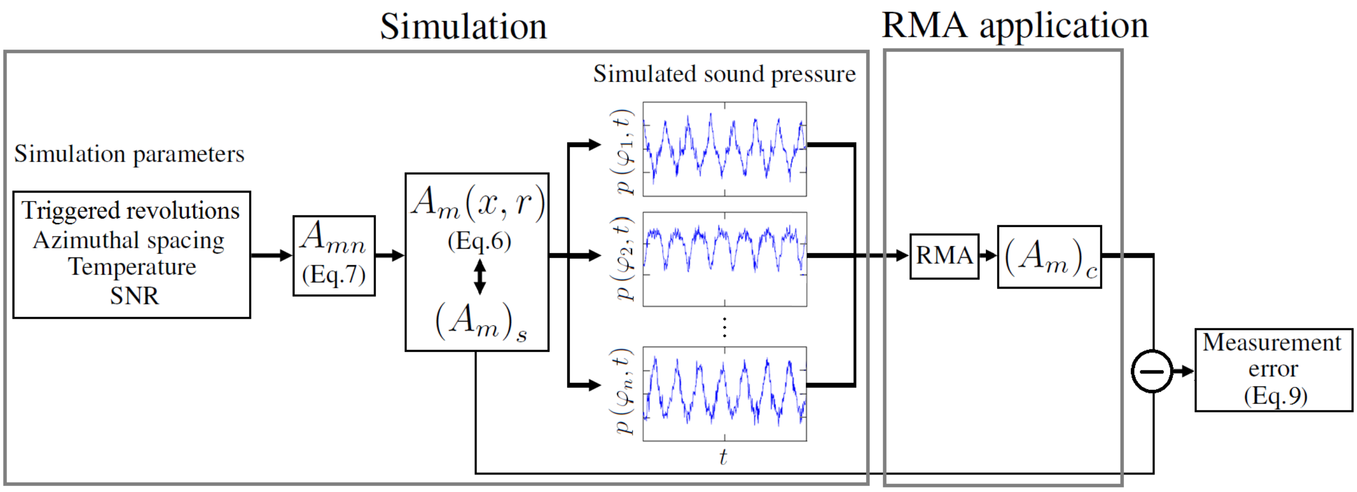

Figure 1: Sensitivity analysis procedure

RESULTS

In order to examine the effect of data acquisition and operating parameters on the RMA for sound propagation studies, a 300 kW single-stage low pressure air turbine (LPT) is considered as test object for the sensitivity analysis. The RMA is applied based on the operating and geometrical data of the LPT, shown in Figure 2 in a two-stage configuration. The present single-stage air turbine features a cylindrical section downstream of the stage, followed by an annular diffuser which further downstream discharges the flow vertically downward into a pipe. Some relevant operating parameters of the turbine which are required for the model are presented in Table 1.

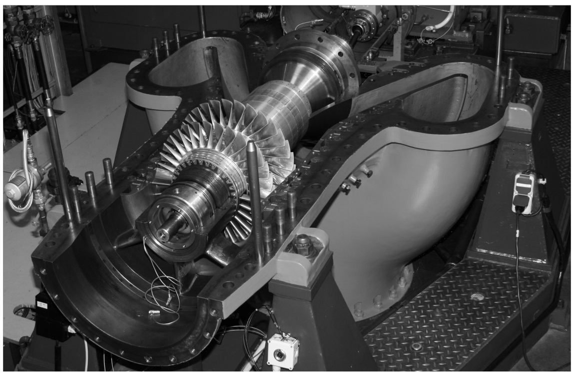

Figure 2: Air turbine

Inlet Mach number	0.215
Inlet total temperature T_e	100°C
Rotor Blades	30
Stator Vanes	29
Nominal angular speed n	7500 min^{-1}
Blade Passing Frequency, BPF	3750 Hz
Downstream section outer radius, R	238 mm

Table 1: Operating data - Air Turbine

As previously indicated, the simulation requires the generation of sound pressure signals at several axial and circumferential positions. Consequently, a rotating ring is designed within the framework of the present research for this purpose (Fig. 3). The simulation generates the sound pressure measurements that the rotating unit would acquire based on the methodology presented before. The ring is to be located in the cylindrical section approx. five cord lengths downstream of the blades. It contains six axially flush mounted 1/4"-special high temperature pressure field microphones from the company G.R.A.S®. The axial spacing between microphones is 20 mm (Fig. 3). The microphones are to be installed in a way that they may be rotated 360° in circumferential direction. The rotation is accomplished by a geared ring coupled to a transmission gear, providing a complete rotation with a resolution as low as 0.1°. This would lead to a theoretical number of 2160 combined axial and azimuthal measurement positions.

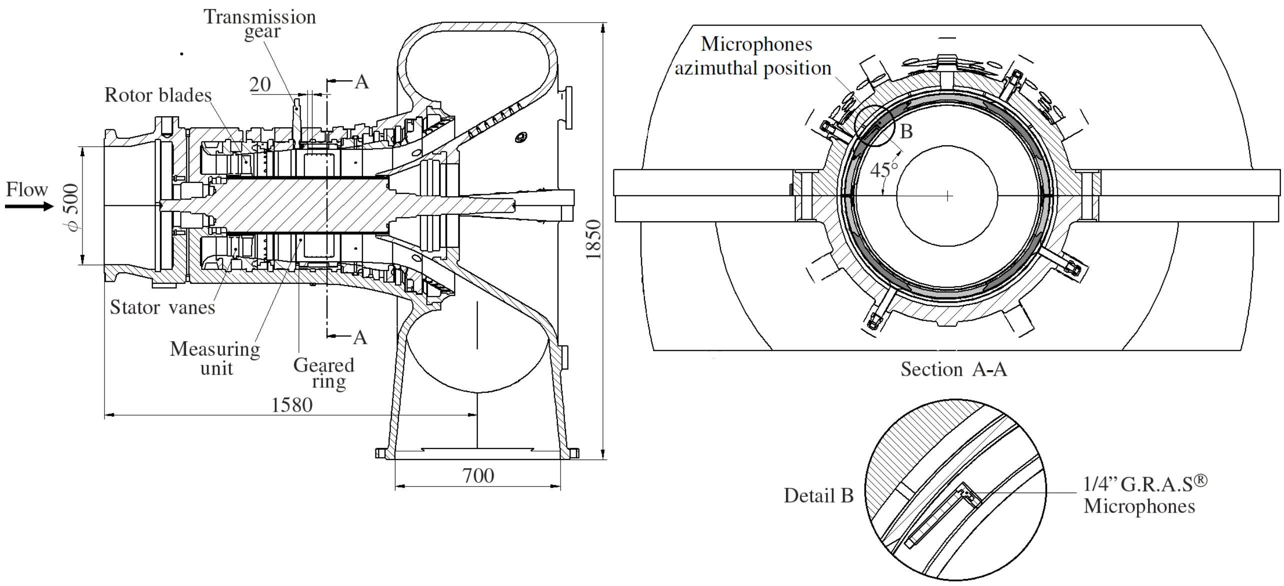

Figure 3: Schematic overview of the air turbine (left), sectional view of the measuring unit and detail view of the microphones (right), (dimensions in millimeters).

The expected propagating mode combinations for the BPF and two harmonics of this frequency are the following: for the BPF, the mode (m, n) = (1,2) is dominant. The first harmonic of the BPF propagates with the mode combinations (-27,0) and (2,4). An additional mode is present at the second harmonic of the BPF, with the combinations (-26,3), (3,6) and (32,2). Based on this information, the ranges of the simulation parameters for the sensitivity analysis are determined (Table 2). The Mach number and duct radius are kept constant, while the temperature in the duct is varied from 80° to 100°C. Within this real operating temperature range for the single-stage turbine used as test object for the present study, the mode combinations (m, n) for each BPF and its harmonics are cut-on. The angular resolution is kept below 5° to assure that the number of azimuthal data acquisition points are at least twice the azimuthal mode order for a proper spatial Fourier analysis. The considered range for the signal-to-noise ratio (which represents the fraction of background noise superposed on a measured signal) is typical for measurement microphones. The parameters SNR, N, and $\Delta\varphi$ are varied such that the RMA results are influenced without altering the modal structure. The sensitivity analysis is done only for the propagating modes derived from the estimated modal structure within the air turbine, i.e. the mode combinations considered correspond to those previously specified.

Table 2: Ranges of the simulated parameters

Azimuthal spacing, $\Delta\varphi$	1°...5°
Operation temperature, T	80°...100°C
SNR	10...60 dB
Number of triggered revolutions, N	1...100

Influence of the sensor position and angular resolution

The influence of the SNR and the angular resolution on the accuracy of the RMA results is studied first. A high quality sensor is expected to have a high SNR, therefore the range of SNR is varied between 10 and 60 dB in intervals of 10 dB, while the angular resolution is resolved in steps of 1°. The measurement time is restricted to one triggered revolution. The operating temperature is kept constant at 100°C.

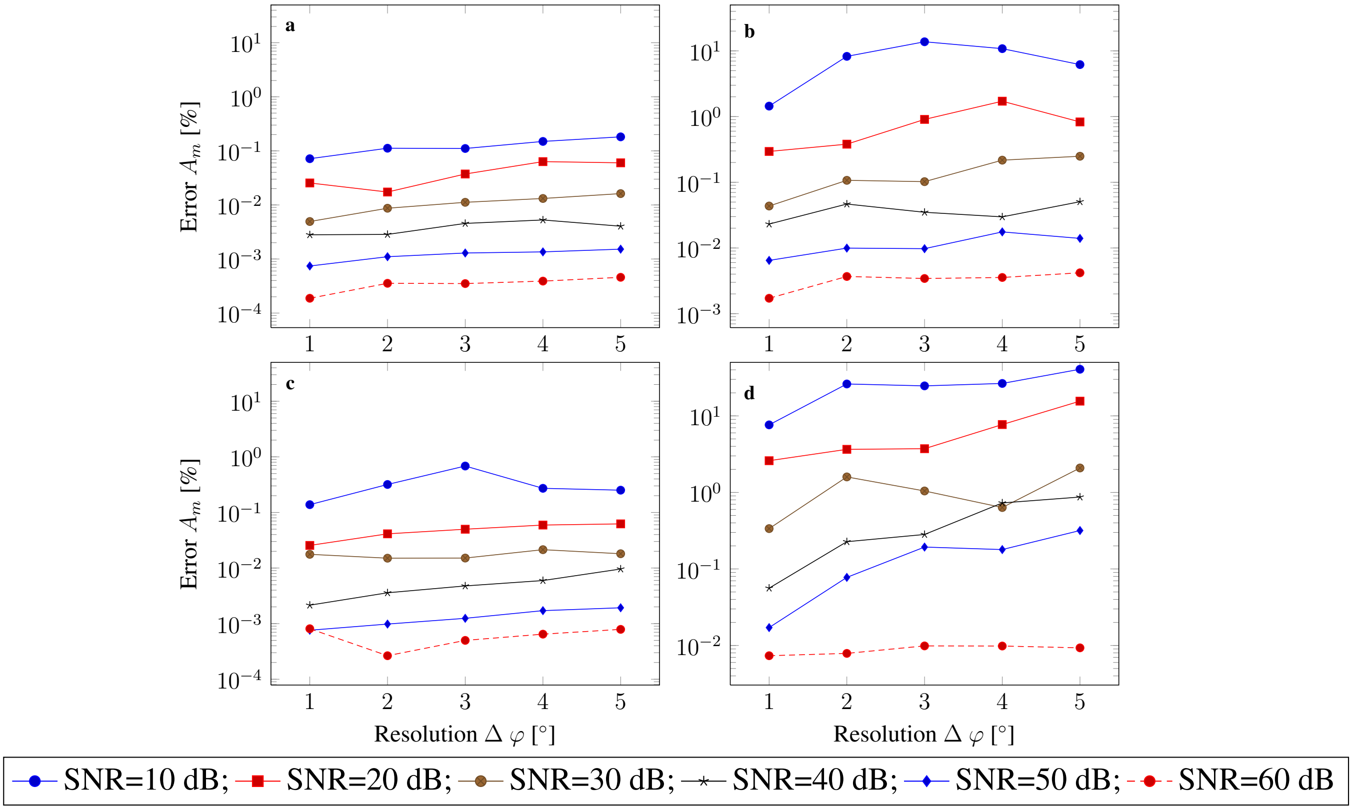

Figure 4: Relative error as a function of the sensor Signal-to-Noise Ratio (SNR) and the measurement angular resolution for the mode combinations $(m, n) = (1, 2)$ (a), $(-27, 0)$ (b), $(2, 4)$ (c), and $(-26, 3)$ (d).

The influence of the SNR over the results of the RMA is established as follows: Once the simulated sound pressure signal for each measurement position is generated (Eq.8), its power P_S is calculated. The signal power turns to be the squared Root-Mean-Value (RMS) of the simulated signal p_S (Eq.8. Afterwards, noise is added to the original simulated signal. This is done keeping in mind that the SNR represents the ratio of the signal power P_S to the noise power P_N. Therefore, the SNR indirectly influences the signal quality through the noise power, ie., noise amplitude as follows:

$$P_N = \sqrt{P_S \cdot 10^{\left(\frac{SNR}{10}\right)}} \tag{10}$$

The results for chosen mode combinations are shown in Figure 4. The relative error rises with decreasing angular resolution while remaining low for SNR greater than 20 dB, except for the high-order mode (-26,3), with an error of approx. 15%. Low-order modes (1,2 and 2,4) display relative errors below 1% despite the reduced spatial measurement resolution. In contrast to this, for high-order azimuthal mode combinations (-27,0 and -26,3) deviations of more than 40% are observed for SNR between 10 and 20 dB and reduced angular resolution (Figs. 4b and 4d). However, an increase in the SNR leads to an acceptable output (low error in amplitude) for all compared mode orders, regardless of the angular resolution. This result is important because it indicates that the measurement setup may be simplified. High-order modes may be measured without requiring a high circumferential measurement resolution, this as a result of low measurement errors for low angular resolution. The consequence of this, is that the time required to complete a measurement campaign may be reduced by a factor of 5, from the initial 360 to a final quantity of 72 measuring positions, i.e. reducing the angular resolution from 1° to 5°. This reduction is possible if sensors with a SNR greater than 30 dB are employed. In this way, the expected measurement error of maximal 1% may be accomplished within an actual measurement campaign.

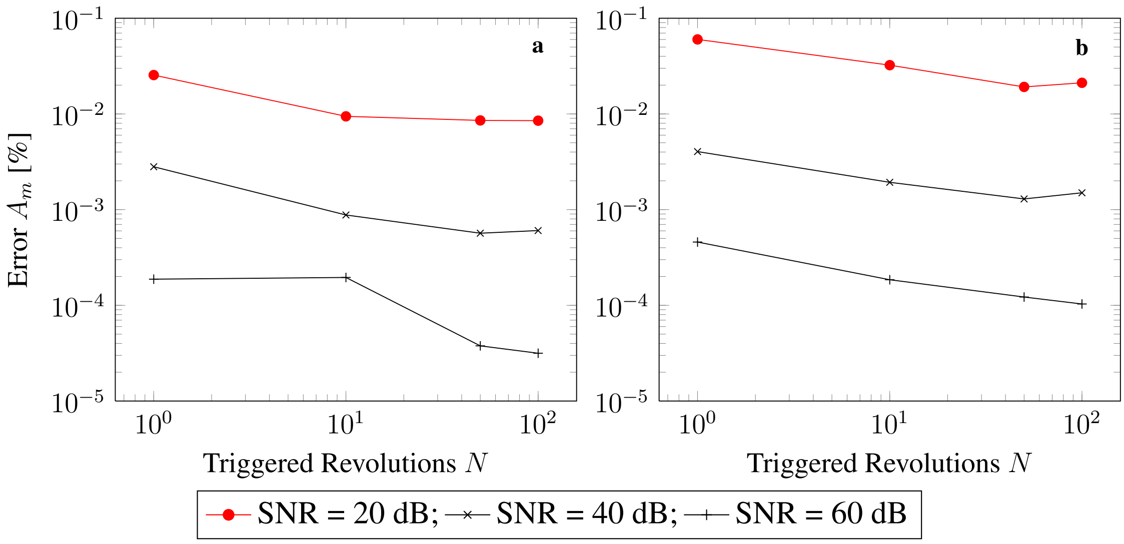

Figure 5: Relative error of mode combination $(1, 2)$ for for $\Delta\varphi = 1°$ (a), and $\Delta\varphi = 5°$ (b).

Angular resolution and influence of the triggered revolutions

The synchronized sound pressure measurements provide a database for the identification of the periodic propagating acoustical modes and a later waveform averaging of the reconstructed circumferential pressure signals. To study the error, sound pressure fields are analyzed over a specific number of rotor revolutions. Thus the influence of the data acquisition periods on the accuracy of the RMA results is studied. A range of measurement periods varying between 1 and 100 revolutions is evaluated while maintaining the angular resolution constant. The influence of the sensor quality is also considered, by varying the SNR from 20 to 60 dB in steps of 20 dB. The operating temperature is kept constant at 100°C. Characteristic results for low and high-mode combinations are shown in Figs. 5 and 6. For low-oder modes (Fig.5a and 5b), the relative error gradually decreases as more revolutions are considered for signal averaging. An increase of almost an order of magnitude in the relative error becomes apparent for decreasing angular resolution, although the error remains below 0.1%. The influence of the SNR on the measurement error is confirmed, as an increase in SNR assures lower errors. Higher modal orders (Fig. 6b) lead to errors above 1% for low SNR independent of the angular resolution. An improvement in measurement error is detected for data acquisition periods of more than one revolution. Above 10 revolutions, the error tends to stabilize and remains almost unchanged for acquisition of additional periods. This is due to the waveform averaging, in which nonstationary tones and broad-band noise are separated from the tonal components caused by the rotor blades passing the stator vanes.

Angular resolution and temperature influence

Typical temperature variations of the flow field behind the single stage configuration of the air turbine are considered. The impact of these temperature variations on the RMA accuracy during a measurement is investigated. The angular resolution is varied in steps of $1°$. The measurement time is restricted to ten triggered revolutions and the SNR is chosen to be 60 dB. These parameters guarantee minimal measurement errors, according to the previously presented results. Fig. 7 shows the relative error as a function of three turbine operating temperatures and the measurement angular resolution for the mode combinations (1,2), (-27,0), (2,4) and (-26,3). Although deviations arise for the whole range of angular resolutions, from $1°$ to $5°$, their magnitude is sufficiently low, with peak values in the order of $10^{-3}\%$, except for the mode (-26,3), where peak values of $10^{-2}\%$ are reached. Accordingly, temperature variations do not affect the measurement error. It's variation may be attributed to the random noise added to the signal during the simulation through the noise power and a chosen SNR.

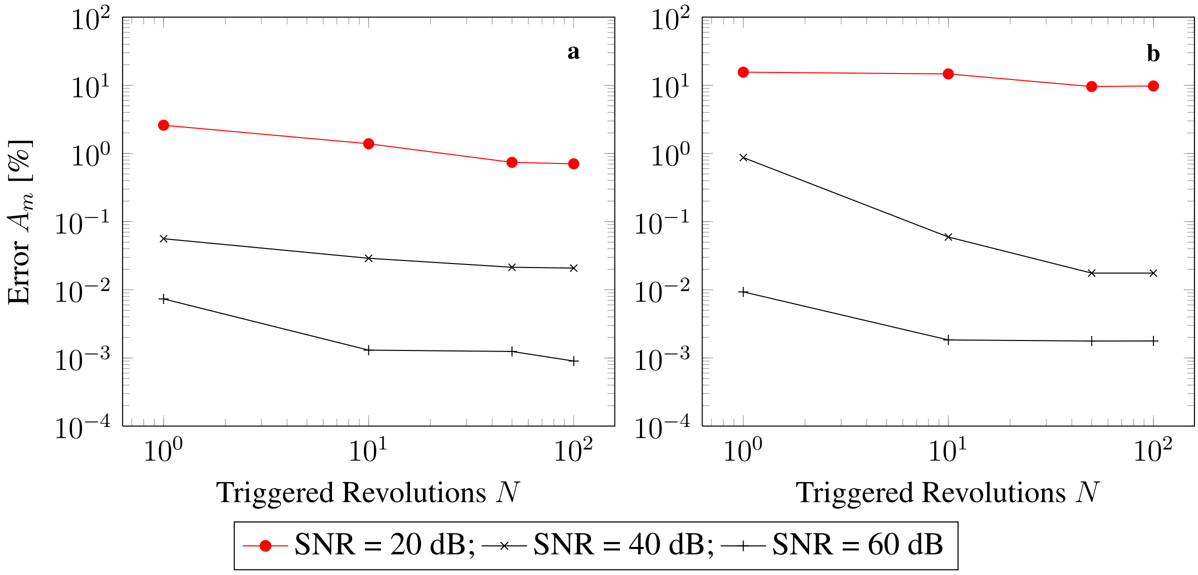

Figure 6: Relative error of mode combination $(-26, 3)$ for $\Delta\varphi = 1°$ (a), and $\Delta\varphi = 5°$ (b).

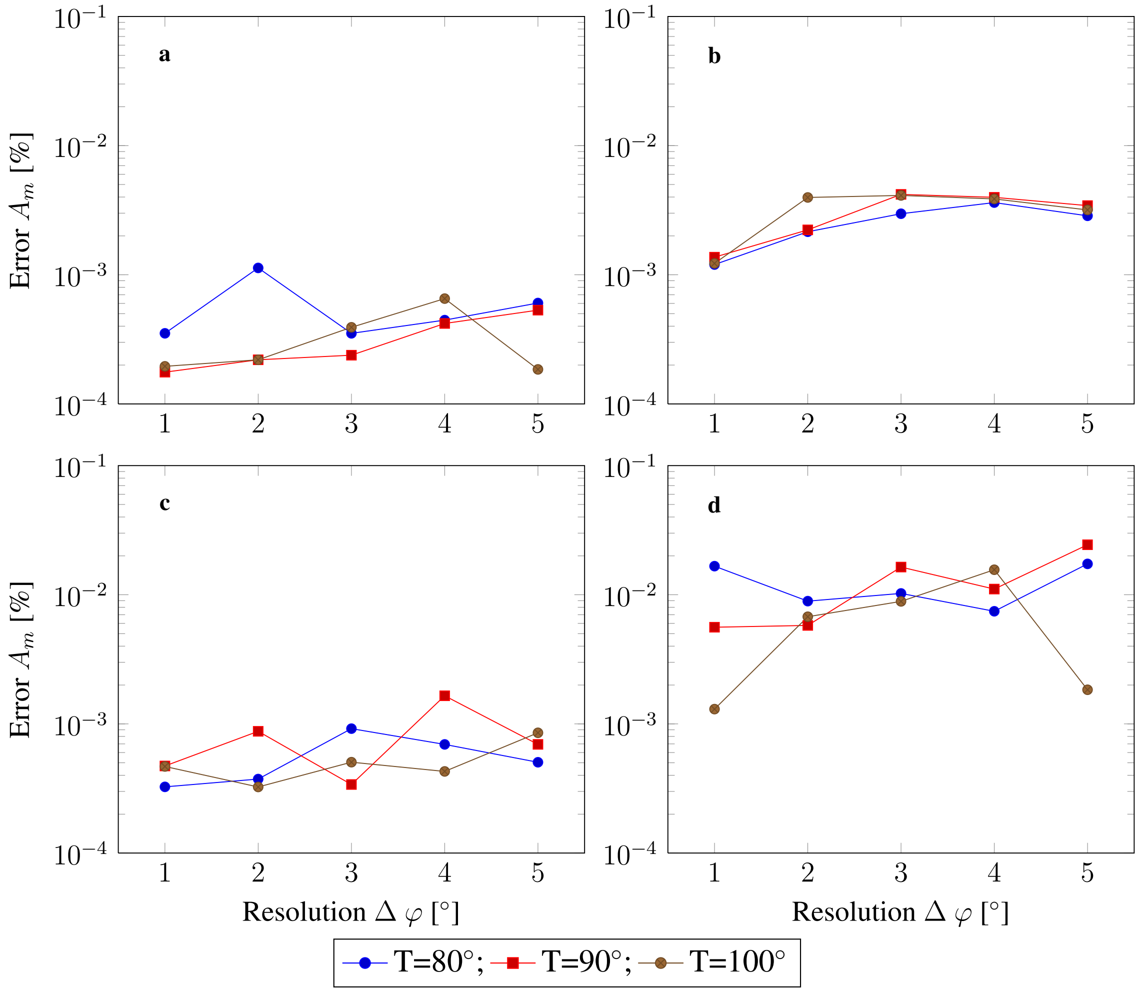

Figure 7: Relative error as a function of the turbine operating temperature and the measurement angular resolution for the mode combinations $(m, n) = (1, 2)$ (a), $(-27, 0)$ (b), $(2, 4)$ (c), and $(-26, 3)$ (d).

CONCLUSIONS

The influence of data acquisition and operating parameters on the output of the Radial Mode Analysis (RMA) to be carried out on a 300 kW one-stage air turbine is assessed by means of a sensitivity analysis. The analyzed parameters include one sensor-related variable, specifically, the signal-to-noise ratio, SNR. Two data acquisition variables are taken into account, being the number of triggered revolutions over which sound pressure data is sampled and the number of measurement positions in terms of the azimuthal spacing of sensors. Machine-related operating parameters are also considered through the temperature variation within the air turbine. The present sensitivity analysis rests on the generation of a typical acoustical modal structure within a turbomachine according to actual turbine-specific operating data. Based on this modal structure, sound pressure signals are simulated and then analyzed with the RMA. Both signals, simulated and analyzed, are compared in order to establish a relative error in the amplitudes of the azimuthal modes.

Generally, a reduced number of circumferential measurement positions (72 azimuthal positions corresponding to an angular resolution $\Delta\varphi = 5°$) assure sufficiently good results for low order modes ([1,2] and [2,4]) as reflected by the relative errors lying below 1%. Better results are expected provided the SNR is high enough. Differences of almost 3 orders of magnitude in the relative error are identified for a SNR varying between 10 and 60 dB, being close to 10^{-4}% for SNR = 60 dB. This is especially important for high-order modes ([-27,0] and [-26,3]), where considerable measurement errors (above 10%) are identified for low signal-to-noise ratios.

Regardless of the SNR, the relative error for low as well as for high-order modes remains almost constant, varying only as a result of the SNR. Consequently, a measurement campaign wouldn't necessarily require fine-spaced circumferential measurement positions for satisfactory results. On the other hand, a reasonably low number of triggered revolutions ensures optimal results independent of the SNR for low-order modes. According to this, no more than ten triggered revolutions are required to assure a low relative error, being this below 10^{-2}% for mode [1,2] considering a SNR of 20 dB. In contrast to this situation, high relative errors (above 10%) appear as high-order modes are considered, although an increase of the SNR results in a considerable decrease of the relative error even for a low number of triggered revolutions (10 triggered revolutions result in an error of 10^{-3}% for a SNR of 60 dB). Finally, the turbine-operating temperature doesn't greatly affect the relative error for a SNR of 60 dB and more than 10 triggered revolutions, being below 10^{-1}% regardeless of the angular resolution and temperature.

The results confirm that it is possible to resolve high-order modes keeping the measurement error below 1% by measuring at a reduced number of circumferential positions - as low as 72 instead of 360 azimuthal positions. This is possible if sensors with high enough signal-to-noise ratios are employed - ideally 60 dB - for a number of more than one measurement period - ideally ten. Being this not always the case for a realistic measurement setting, where a lower signal-to-noise ratio sensor may be employed (SNR = 20 dB), a circumferential resolution of 5° as well as at least 100 triggered revolutions are recommended to keep the relative error below 10%.

In future work, the consequences of inaccurate triggering could be investigated. By defining a maximum trigger delay and computing the mean of several generated signals, the influence of the averaging on the calculated amplitudes of the measured signal can be determined. The accuracy of the determination of the azimuthal modes and their amplitudes would additionally be influenced by different trigger delays at separate azimuthal positions. Evaluating these simulations in addition to the present ones, extensive knowledge may be gained regarding the errors inflicted on the modal analysis by imperfect measurements. This knowledge can be used to define requirements for good measurements and will help improve future setups.

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