ROTATIONAL SPEED OF A DAMAGED FAN OPERATING AT WINDMILL

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ABSTRACT
Following an engine shut-down event due to the loss of a fan blade, the fan rotates in an unbalanced windmill condition leading to undesirable structural excitation. The flow through a fan stage containing idealised damaged rotors is investigated numerically at the windmill condition. Specifically, predictions of rotational speed and flow are presented, which agree with undamaged engine scale test data, and the governing rotor flow features are understood. Results of an undamaged rotor show the tip and hub sections operate as a turbine and compressor respectively, with small flow deviation from the geometric exit angle. Introducing axisymmetric tip damage acts to remove blade sections that induce high positive whirl thereby reducing turbine work extraction, and in turn rotational speed. Non-axisymmetric tip damage is then considered to understand the impact of rotor-to-rotor flow interaction, between blades of different damage levels, upon the rotational speed. In all instances the non-axisymmetric rotational speed is higher than the average of the component axisymmetric speeds.

NOMENCLATURE
Symbols
A  Area
\( c \)  Absolute velocity
\( C_P \)  Constant pressure specific heat
\( C_D \)  Discharge coefficient
\( m \)  Mass flow rate
M  Mach number
P  Pressure
r  Radius
s  Pitch
T  Temperature
U  Blade speed
\( w \)  Relative velocity
\( W \)  Corrected flow
\( W^* \)  Isentropic corrected flow

\[ \Omega \]  Rotational speed

Acronyms & Abbreviations
AA  Axisymmetric average
ESS  Engine section stator
NPR  Nozzle pressure ratio
OGV  Outlet guide vane
R  Rotor
Un.  Undamaged

Subscripts
amb  Ambient
fc  Fan case
ni  Nozzle inlet
T  Total quantity
x  Axial direction
0  Domain inlet quantity
1  Rotor inlet quantity
2  Rotor exit quantity
3  OGV inlet quantity
4  OGV exit quantity
5  Bypass domain exit quantity
6  Core domain exit quantity
25/Un%  Non-axi. domain
s1-5  Simulation 1-5

Greek Symbols
\( \alpha \)  Absolute flow angle
\( \beta \)  Relative flow angle
\( \delta \)  Flow deviation angle
\( \Delta \)  Total pressure loss
\( \chi \)  Metal angle
\( \tau_s \)  Shear stress
\( \psi \)  Work coefficient
\( \omega \)  Vorticity
INTRODUCTION

Free windmilling (zero-torque) of the fan occurs after an engine shutdown event when the inlet ram pressure, due to the forward speed of the aircraft, maintains spool rotation (Walsh & Fletcher 2005). An engine shutdown event can occur with or without damage to the fan, following a combustor flame out or fan blade off event for example. Particularly during assessment of damaged windmilling, consideration of the fan rotational speed and associated unbalance is important when quantifying the frequency and magnitude of excitation transferred to the aircraft wing structure. This paper presents a numerical study on the windmill rotational speed of a damaged fan and provides physical understanding of the flow mechanisms which influence this parameter relative to an undamaged baseline.

Undamaged windmilling has been considered extensively. Prasad & Lord (2010) present global measurements of a low hub-to-tip ratio fan stage, at full engine scale. Garcia Rosa (2014) and Ortolan et al. (2016) provide radial profile measurements at axial stations throughout a fan stage rig. In all studies, a zero-torque characteristic is defined by a linear relationship between non-dimensional rotational speed and flow, which Gunn & Hall (2015) demonstrate analytically, is solely dependent on relative flow turning.

Insight into the flow field of high bypass ratio transonic fan stages is explored numerically by Prasad & Lord (2010), Dufour et al. (2015), Gunn & Hall (2015), and Ortolan et al. (2016). The rotor hub and tip are seen to operate as a compressor and turbine respectively, which effectively cancel out to maintain zero work. Gunn & Hall (2015) show the rotor work distribution is strongly influenced by downstream blockage in the core and bypass stators, which are shown to experience strong negative incidence leading to fully detached flow across their concave surfaces.

Similarly, the rotor is also subject to negative flow incidence. All accounts describe a highly three-dimensional rotor separation zone, which for the purpose of this study, is important to understand as it strongly influences mass flow, work distribution, and in turn the rotational speed. Gunn & Hall (2015) investigate further and flow streamlines indicate the presence of a strong leading edge vortex, which migrates radially towards the tip. Although not explicitly stated, the separated region is seemingly governed by the vortex diameter, which grows towards the tip. Dufour et. al (2015) also identifies the leading edge vortex and states, unlike rotating stall, shedding is stabilised by the concave surface - implying camber distribution plays an important role in governing the rotor separation. Gill et al. (2010) and Goto et al. (2014) notice similar unsteady vortical phenomenon, albeit on high hub to tip ratio compressor blades.

In addition to the mass flow distribution, the flow turning influences the radial work distribution. A velocity triangle diagram is shown in Figure 1 comparing the design and windmill operating points. Relative to the design point, the windmill regime is characterised by high flow coefficient and low blade speed (Gunn & Hall 2015). The velocity triangle depicts a turbine acting tip section operating with a negative absolute swirl angle at the rotor exit. In the relative frame, Prasad & Lord (2010) and Gunn & Hall (2015) observe small deviation, δ, between relative whirl angle, β2, and the metal angle, χ2. Consequently it is clear from Figure 1, that blade designs with higher tip χ2 extract more work which, as shown, leads to a higher rotational speed.

In summary, literature covering the undamaged condition suggests the rotor geometry governing the windmill rotational speed is; tip clearance, χ2, and camber distribution. As a result, this paper addresses idealised tip and trailing edge damage, which vary these parameters from design intent.

Figure 1 illustrates trailing edge damage effectively increases the metal exit angle by Δχ2. Figure 2 illustrates the sequence of events following an isolated increase to χ2, whether by design or via damage, to the windmill rotational speed. Consider arbitrary tip and hub section velocity triangles of a fan operating at a baseline windmill condition where positive compressive work, ψc, equals negative turbine work, ψT. Immediately after χ2 is increased, ψT increases whilst ψc remains constant resulting in an unbalanced negative torque condition. As a result, rotational speed must increase, which in turn acts to reduce the turbine work and increase the compressive work until a new zero-torque equilibrium is restored. Similar logic can be applied to
understanding tip damage when a positive torque imbalance is created by complete elimination of turbine sections.

Figure 1: Velocity triangle diagram comparing design point and windmill conditions

![Velocity triangle diagram comparing design point and windmill conditions](image)

**Figure 1:** Velocity triangle diagram comparing design point and windmill conditions

<table>
<thead>
<tr>
<th>Turbine Acting Section (Tip)</th>
<th>Compressor Acting Section (Hub)</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Diagram" /></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
</tbody>
</table>

**Figure 2:** Velocity triangle diagrams at tip and hub sections illustrating a change in exit metal angle, \( \chi_2 \)

Realistically, following the loss of a blade, the fan damage profile is highly complex and non-axisymmetric. However, to gain a fundamental understanding, a damaged blade is represented by idealised geometry, and is initially considered axisymmetric around the annulus. Non-axisymmetric tip damage is then investigated through multi-passage simulation with the purpose of identifying the significance of rotor-to-rotor interaction upon the rotational speed. It is evident from Figure 1 that the pitch, \( s \), to chord ratio of adjacent undamaged blades increases, which is expected to strongly influence the passage work distribution.
Furthermore, a change in prediction methodology is required when moving from undamaged to damaged windmilling. Existing windmill numerical predictions are reliant on availability of the zero-torque boundary conditions; fan rotational speed and flow typically deduced via experiment. Alternatively, low order tools can pro – and do so successfully for undamaged windmilling (Prasad & Lord 2010). However, without an empirical reference, predictive capability deteriorates due to flow complexities introduced in a damaged fan stage. Consequently, this paper presents an iterative technique of the boundary conditions using the zero-torque characteristic.

NUMERICAL METHODOLOGY

Domain & Test Case Rationale

The numerical domain is shown in Figure 3 and consists of a rotor, outlet guide vane (OGV) and engine section stator (ESS). Whilst not directly analysed, stators are included in the domain to create the required downstream blockage to maintain representative radial mass flow distribution across the rotor. Axial stations at the domain inlet, upstream and downstream of the rotor and stators, and exit are indicated. Two variations of wide-chord, low hub-to-tip ratio rotors are explored and will be referred to as Rotor A and Rotor B. Rotor A is designed with a higher positive \( \chi_2 \) radial distribution in the bypass region.

A summary of the test cases is presented in Table 1. This study commences with an undamaged, Un., comparison of the two rotor standards operating at windmill. Rotor B is then chosen to explore idealised “tip” and trailing edge “step” damage types, which are illustrated overlaid in Figure 3. The tip gap of Rotor B varies from 5%, 15% to 25% of the undamaged span. The stepped case employs a 25% and 50% tip gap to the leading 75% and trailing 25% true chord respectively. Two and four-passage domains, containing rotors of different tip damage levels, are selected to analyse non-axisymmetric effects. The nomenclature used to describe the multi-passage domain is defined as “upstream / intermediate / downstream blade”, and is repeated periodically.

When comparing solutions of different damage, two approaches are chosen to define the operating condition; constant Mach number and constant corrected flow. The former is representative of actual engine operation and utilizes knowledge of the engine geometric nozzle area \( A_{geom\_eng} \) and the nozzle discharge characteristic to define the flow rate at a given flight speed. In this study, \( M = 0.40, 0.63 \) and \( 0.80 \) at a typical cruise altitude is considered. Rotor B is installed in an engine with a 12% lower \( A_{geom\_eng} \). Following this approach, iteration of the flow and rotational speed boundary conditions to the zero-torque point is required, which is detailed shortly. Constant corrected flow is predominantly used in this paper to allow an academic consistency when comparing the solution flow fields.

![Figure 3: Rotor B fan stage domain with damaged rotor profiles overlaid](image-url)
Table 1: Test cases

<table>
<thead>
<tr>
<th>Rotor Type</th>
<th>$\Delta_{\text{geom, eng}}$</th>
<th>Domain</th>
<th>Damage Profile</th>
<th>Mesh Cell Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor A</td>
<td>Baseline</td>
<td>1 R, 1 OGV, 1 ESS</td>
<td>Un, Un</td>
<td>4.1 x 10^6</td>
</tr>
<tr>
<td>Rotor B</td>
<td>-12%</td>
<td>2 R, 1 OGV, 1 ESS</td>
<td>5% Tip, 15% Tip, 25% Tip, 25% Step</td>
<td>4.8 x 10^6, 5.6 x 10^6, 6.4 x 10^6, 7.3 x 10^6</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4 R, 1 OGV, 1 ESS</td>
<td>25/Un% Tip, 25/5% Tip, 25/25/Un/Un% Tip</td>
<td>10.9 x 10^6, 11.7 x 10^6, 19.0 x 10^6, 21.6 x 10^6</td>
</tr>
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</table>

**Computational Setup**

Three-dimensional steady RANS simulations were employed across the test cases using the Rolls-Royce plc. code, HYDRA. Closure is achieved through the Spalart-Allmaras turbulence model (Spalart & Allmaras 1994), where $y^+$ is consistently less than 2.5 across all viscous wall boundaries. Despite the expectation of highly separated flow, full unsteady calculation was discarded due to high computational requirements. Dufour et al. (2014) and Ortolan et al. (2016) find steady RANS provides a reasonable approximation across the rotor, which remains the focus of this study in predicting rotational speed trends with damage. Mixing planes are applied at the interfaces between the rotor and stator domains. It is appreciated that circumferential flow averaging presents an unphysical stator inlet condition, particularly when sampled over multiple passages. However, since the stator flow field is not focus of this study, a mixing plane approach is sufficient to provide an adequate representation of the upstream flow blockage, and in turn rotor work distribution for prediction of rotational speed.

A structured multi-block grid topology is automatically generated using Rolls-Royce’s code, PADRAM (Shahpar & Lapworth 2003). A front view of the 25/Un/Un/Un% tip grid and a meridional view of the 25% tip is presented in Figure 4a) and b) respectively. Damage is modelled through chordwise control of tip clearance during the grid generation phase. The tip surface is concentrically interpolated between the casing line and the nearest throughflow streamline to ensure a consistent tip gap. O-block meshes are used to encompass the blade sections (239 sectional elements), and H-blocks (120 radial elements) are used to fill the remaining annulus. As illustrated in Figure 4b), an additional H-block is used to fill the regions vacated by tip and step damage. In this important region, a grid independency study is completed where the number of radial elements is doubled until the rotational speed and flow is converged (< 0.5% variation). 6 cells radially per percentage tip damage were found to be adequate. Cell clustering surrounding the tip propagates
axially downstream to capture the streamwise vortex core and entrainment of tip flow. In multi-
passage domains, tip clustering is also propagated circumferentially onto successive blades to
capture rotor-to-rotor effects, as shown in Figure 4a).

**Zero-Torque Boundary Conditions**

This section describes the iterative process required to deduce the windmill boundary conditions
(Eqn. 1); corrected flow, \( W_5 \), and rotational speed, \( U \), for a prescribed flight altitude and Mach
number – representative of actual engine operation.

\[
W_5 = \frac{m \sqrt{T_T s}}{p_T s} \quad U = \frac{u_{tip}}{\frac{r_{T,1}}{T_amb}}
\]  

(Eqn. 1)

Due to high core blockage at windmill (Walsh & Fletcher 2005), core corrected flow, \( W_6 \), is defi-

dined to maintain a constant bypass ratio of 50:1. Due to high core blockage across the ESS, a typical
windmill bypass ratio is 75–80:1, however due to simulation convergence issues a higher bypass
ratio could not be calculated.

Convergence of the windmill boundary conditions is achieved via an iterative series of five
simulations around the linear zero-torque fan characteristic, illustrated in Figure 5. Above
(Simulation 1) and below (Simulation 2) the characteristic represent net compressive and turbine
work respectively, and thus departure from the windmill condition. In these regions, the fan
operates in an unbalanced torque state, similar to what is described in Figure 2, and physically seeks
equilibrium on the zero torque line. Numerically, iteration to the zero-torque characteristic
(Simulation 3) is achieved by simple linear interpolation at constant speed. A numerical windmill
work coefficient, \( \Psi \), threshold is defined in Eqn 2.

\[
\Psi = \frac{C_p(T_{T,1} - T_{T,2})}{u_{tip}^2} < 10^{-4}
\]  

(Eqn. 2)

Simulation 3 exists on the zero-torque characteristic and represents an arbitrary windmill point.
Movement up the line to Simulation 4 is synonymous to an increase in flight Mach or nozzle area,
\( A_{geom} \). Hence for a given flight condition, a fixed area nozzle engine maintains a unique point on the
zero-torque characteristic, which is identified by the condition \( A_{geom} = A_{geom_eng} \) (Simulation 5) and
is the objective of the iteration. Simulation 3 and 4 nozzle areas; \( A_{geom,s3} \) and \( A_{geom,s4} \), are calculated
by first approximating the nozzle pressure ratio, NPR (Eqn. 3). Fan case, \( \Delta_{fc} \), and bypass duct, \( \Delta_{bp} \),
total pressure losses are assumed constant with Mach number, since they are an order of magnitude
smaller than expected across the fan stage

\[
NPR = \frac{p_{T,ni}}{p_{amb}} \approx \frac{p_T s(1 - \Delta_{fc} - \Delta_{bp})}{p_{amb}}
\]  

(Eqn. 3)

Assuming isentropic flow through the nozzle, the NPR is used in a Q-curve calculation to
deduce the non-dimensional flow through the nozzle, \( W^* \) (Eqn. 4). Since the rotor is operating at a
zero-work condition, total temperature through the bypass stream is considered equal to the ambient
temperature. The reference area is defined at the nozzle exit plane and is specified to account for
blockage due to boundary layer formation.

\[
W^* = \frac{m \sqrt{T_{amb}}}{A_{ref} p_T s(1 - \Delta_{fc} - \Delta_{bp})}
\]  

(Eqn. 4)
Blockage in the nozzle is defined by the discharge coefficient, $C_D$. For this study, experimental data was available, although studies such as Chen et al. (2013) predict nozzle characteristics computationally.

$$C_D = \frac{m_{ref}}{m_{geom}} = \frac{A_{ref}}{A_{geom}} \quad \text{(Eqn 5)}$$

Hence, by substituting Eqn. 5 into Eqn. 4, and solving for the geometric nozzle area, $A_{geom}$ (Eqn. 6), a comparison to $A_{geom\_eng}$ can be made.

$$A_{geom} = \frac{m_{\sqrt{T_{amb}}}}{C_D W^* P_{T5}(1 - \Delta_{fc} - \Delta_{bp})} \quad \text{(Eqn. 6)}$$

Simulation 5 is setup following a linear interpolation of the rotational speed and corrected bypass flow, between Simulations 3 and 4, to coincide $A_{geom} = A_{geom\_eng}$. For all cases concerning Rotor A and B, $A_{geom\_s5}$ is within 1% of $A_{geom\_eng}$ and is robust to the initial guess provided by the low order tool.

**Figure 5: Windmill operating point prediction process**

**AXISYMMETRIC RESULTS**

The zero-torque characteristics are shown in Figure 6 for axisymmetric undamaged and damaged cases. Contours of constant Mach number are presented for the Rotor B tip damaged cases, which are calculated using the iterative process previously mentioned. In addition, Rotor B damaged cases are predicted at the undamaged corrected flow at $M = 0.40, 0.63,$ and $0.80$ ($W_1 = 0.07353, 0.1028$, and $0.1178$ respectively). At this flow, the rotational speed was iterated until the operating condition satisfied the work coefficient threshold defined in Eqn. 2.

Undamaged corrected mass flow and rotational speed experimental data is available for Rotor A, which is measured in an altitude test facility at representative flight conditions. Good agreement of the zero-torque characteristic prediction to measurement is seen. Whilst no experimental data exists for the idealised damaged cases, the validation presented is deemed sufficient because, as is shown later, the flow topology is qualitatively similar to the undamaged case.

**Rotational Speed Performance**

Interpolating to equal flow ($W_1 = 0.07353$), the undamaged Rotor A windmills approximately 10.5% faster than Rotor B at all flight conditions. To understand further, radial distributions at the rotor exit are presented in Figure 7 for the undamaged Rotor A and B, Rotor B 5%, 25% tip and 25% step cases. Relative whirl angle illustrates deviation is similarly small between the two undamaged cases. Referring to Figure 2, due to a higher positive $\chi_2$ between 20-95% span, Rotor A...
extracts a higher turbine work and thus rotates faster. The crossover point between compressive and turbine work, \( r_0 \), is 57%, and is equal for both rotor standards.

As Rotor B tip damage increases, blade sections with high positive whirl or \( \chi_2 \) are removed, which by following logic described in Figure 2, creates a positive torque imbalance. As a result, Figure 6 shows the windmill rotational speed and the slope of the zero-torque characteristic reduces. Total temperature radial profiles are shown in Figure 7a). In the 25% tip freestream, there is no flow turning in the absolute frame and thus zero temperature rise. Peak work extraction in the reformed turbine-acting region of the 25% tip damaged blade is lower than the previously undamaged case, thereby reducing rotational speed. However, a local increase in deviation is observed near the 25% damaged blade tip, which counteracts the reduction in rotational speed. \( r_0 \) now occurs at 47% undamaged span, which equates to 63% of the damaged span, implying with increased tip damage the turbine region is concentrated over a smaller portion of the blade.

At equal flow, the 25% step windmills 9.1% faster than the 25% tip case. Reflected in 6b), removing material from the trailing edge effectively increases \( \chi_2 \), which in similar fashion to the previous analysis, explains the increase in windmill speed.

The axial velocity radial distribution is presented in Figure 7c). For all cases, a significant blockage source is seen in the tip region, which impairs the turbine work extraction. The 5% tip gap case exhibits a unique jet structure, which is characterised by a region of high axial velocity and reduced \( \beta_2 \). Here, the work distribution profile indicates a secondary compressive region in the tip gap, which acts to reduce the rotational speed. A secondary compressive region is also seen on the 25% tip and step cases, although significantly smaller due to the increased tip freestream area.

Readdressing the problem at a global level, Figure 8 depicts a linear relationship between Rotor B rotational speed and damage at \( W_1 = 0.07353, 0.1028 \). This result is intuitive for these particular rotor designs where \( \frac{d\chi_2}{dr} \) is near constant. The reduction in rotational speed is amplified at higher flow – confirmed by diverging zero-torque characteristics shown in Figure 6. These trends can be extrapolated to a point of intersect, which corresponds to a tip damage condition of flow independence at 70.3% span loss. Interestingly, this theoretical condition occurs at zero blade speed. Beyond this point, the rotor is expected to windmill in the opposite direction, since the majority of the remaining span is designed with negative \( \chi_2 \). However, at severe damage linearity is speculative, particularly when the upstream influence of the core stator blockage, seen between 0-30% span (Figure 7b), occupies a larger portion of the damaged rotor span.

**Figure 6: Zero-torque fan characteristic with contours of constant Mach number**
Figure 7: Axisymmetric rotor exit radial profiles a) total temperature rise b) axial velocity c) relative whirl angle (Rotor A: $W_1 = 0.07579$, Rotor B: $W_1 = 0.07353$)

Figure 8: Rotational speed variation with damage (Rotor B)

**Rotor Flow Field**

The rotor flow field is visualised in Figure 9 for the undamaged, 25% tip and step cases at $W_1 = 0.07353$. Flow streamlines are contoured with vorticity, whilst axial shear stress is presented on the concave blade surface to indicate the regions of flow separation. For these cases, axial velocity contours at the rotor exit are shown in Figure 10.

Consistent with Gunn & Hall (2015), the undamaged case exhibits a separation over the leading edge, which rolls into a vortex. Contained by the concave surface, the low momentum fluid radially migrates towards the tip due to the centrifugal loading. The leading edge vortex then rotates into the streamwise direction in the proximity of the casing, and finally propagates downstream. Evidently, the structure of the streamwise vortex accounts for the aforementioned tip blockage, seen in Figure 7b). From Figure 10a), the streamwise vortex is elongated circumferentially and encompasses approximately 50% of the tip passage area.

Comparing Figure 9a) and 9b), increasing tip damage influences the vortical structure. Using the velocity triangle diagram in Figure 1, a damaged rotor windmilling at a reduced blade speed with the same inlet axial velocity experiences further negative incidence. As a result, the leading edge separation zone extends further downstream. Synonymously, the leading edge vortex is predicted to manifest with a larger diameter and experiences a larger count of bulk rotations during its radial migration. Consequently, the leading edge separation zone and vortex diameter are coupled in size. The 25% tip case leading edge vortex gradually transitions into the streamwise vortex at the blade tip, now away from the casing. The high velocity freestream flow convects the
vortex into the streamwise direction, and is entrained to alleviate the blockage of the vortex core, which is illustrated in Figure 7c) and 10b).

Turning attention to the 25% step case, the removal of trailing edge material does not interfere with the leading edge vortex, and thus the separation zone is similar to the 25% tip case. Flow is deflected around the leading vortex and approaches the step with positive incidence. As a result, a secondary streamwise vortex contra-rotates over the step region, which is observed as increased blockage between 50–70% span (Figure 7c). Flow is then redistributed radially towards the mid-span, where a region of high axial velocity is seen in Figure 10c).

Consistent with literature, a trailing edge hub separation arises due to strong radial flow migration created by high core stator blockage. Whilst this has stronger implications on the downstream performance of the splitter and bypass stator, the hub blockage reduces the effectiveness of the compressor acting region, thus increasing rotational speed.

![Figure 9: Flow visualisation (view from concave surface) a) U b) 25% tip c) 25% step (Rotor B: \( W_1 = 0.07353 \)](image)

![Figure 10: Axisymmetric damage Rotor B; rotor exit axial velocity \( \frac{V_{ax}}{V_1} \) a) U b) 25% tip c) 25% step (Rotor B: \( W_1 = 0.07353 \)](image)

**NON-AXISYMMETRIC RESULTS**

The multi-passage windmill condition is defined when the rotor sector work coefficient satisfies the prescribed threshold defined in Eqn. 2. Passage work coefficients are deduced via integration of the blade surface pressure field to avoid collusion from adjacent blades. Figure 11 illustrates that individual blades operate at a non-zero passage work coefficient at the non-axisymmetric windmill condition. Short and long blades operate with net positive compressive and negative turbine work due to an increase and reduction in rotational speed relative to the axisymmetric condition respectively. Upon further investigation of the four-blade cases, the torque contribution is a strong function of circumferential damage sequencing. Notably, when the ratio of undamaged to damaged blades is increased from 1:1 to 3:1, the turbine torque imbalance is shared across the undamaged blade whilst the 25% tip damaged blade torque remains similar.
Rotational Speed Performance

Figure 12 displays the rotational speed of the 25/25/Un/Un%, 25/Un/Un/Un%, 25/5% and 25/Un% tip damaged non-axisymmetric cases at $W_1 = 0.07353$. The multi-passage rotational speed is shown relative to the component axisymmetric speeds and their average at the same flow. The multi-passage speed is always higher than the axisymmetric average (AA) indicating strong rotor-to-rotor interaction. Deviation from the AA is highest in the 25/5% case, where an 11.85% overspeed is observed.

![Figure 11: Multi-passage blade work coefficient (Rotor B: $W_1 = 0.07353$)](image-url)

Rotor Flow Field

The flowfield is investigated to provide a physical understanding of the non-axisymmetric overspeed relative to the AA. Axial velocity contour plots are presented for all non-axisymmetric cases in Figure 13. Radial temperature, relative whirl angle and axial velocity distributions are shown in Figure 14 for the 25/Un% case only as this solution exhibited distinguishable rotor-to-rotor interactions. Within the multi-passage domain, flow quantities are circumferentially averaged across the individual blade passage at axial station 2. For reference, the 25% damaged blade within the 25/Un% non-axisymmetric domain is referred to as 25/25/Un%. The undamaged and 25% axisymmetric cases are recalculated at the 25/Un% non-axisymmetric rotational speed (denoted $U_{\Omega 2}$ and 25/25/Ω%), to account for a change in incidence, and to isolate blade proximity effects.

Analysis of the multi-passage flowfield presents three key rotor-to-rotor interactive features, which influence the rotational speed away from the AA.

Firstly, Figure 13 suggests the size of the undamaged streamwise vortex is a function of the blade damage sequence. When the concave surface of a long blade faces a short blade, the long blade streamwise vortex detaches from the blade tip and grows into the gap. This manifests as a sharp reduction in axial velocity between 80-100% annulus height in the 25/25/Un% passage (Figure 14c). As a result, less flow is turned and the peak turbine work of the $U_{\Omega 2}$ blade reduces relative to the $U_{\Omega 2}$ case - acting to reduce the rotational speed. Comparing the 25/Un%, 25/25/Un/Un% and 25/Un/Un/Un% cases in Figure 13, this effect is mitigated when the convex
surface of the long blade is facing another long blade, which cascades further with multiple upstream undamaged blades.

Secondly, the detached streamwise vortex from the undamaged blade causes flow to redistribute into the 25% passage. Relative to the 25% case, the 25% streamwise vortex is suppressed and tip separation is prevented. As indicated in Figure 14c), the axial velocity is increased across the entire span by ~10%. This is reflected as an increase in short blade turbine work between 50-75% undamaged span.

Finally, from Figure 14a) it is seen that, unlike the 25% case, turbine work is now extracted in the 25% tip freestream. In this region the pitch-to-chord ratio of the adjacent undamaged blade sections effectively double and turbine flow turning extends, albeit attenuated, into the tip freestream. Since the mass flow which is usable for work extraction increases significantly, this effect dominates the overall increase in rotational speed relative to the AA.

CONCLUSIONS
This paper considers the flow through a low hub-to-tip ratio fan stage, containing rotor damage, at the windmill condition. Specifically, the importance of axisymmetric tip and trailing edge, and non-axisymmetric tip damage upon the windmill rotational speed is evaluated numerically. The key findings of the study are:

1) An iterative numerical technique is used to predict the windmill boundary conditions, and shows good agreement with undamaged engine test data.
2) Tip damage acts to remove blade sections of high positive $\chi_2$, thereby reducing work extraction in the reformed turbine region, and in turn rotational speed.
3) In similar fashion, damage to the trailing edge locally increases $\chi_2$, leading to a speed increase relative to blades with pure tip damage.
4) Cases of minor tip damage exhibit a tip gap jet structure characterised by high axial velocity and low relative flow turning. In this region, a secondary compressive zone acts to reduce rotational speed.
5) In all instances the non-axisymmetric rotational speed is higher than the average of the component axisymmetric speeds. This result implies rotor-to-rotor flow mechanisms are relevant and multi-passage domains are mandatory for prediction of rotational speed. The dominating mechanism is distinguishable when a short blade is positioned between two long blades. Here, the pitch-to-chord ratio of adjacent sections effectively double and flow turning extends, albeit attenuated, into the tip gap contributing to higher passage turbine work.

It is the scope of future work to apply the presented methodology on a larger database of damage combinations and types from which a statistical analysis can be conducted. It is of particular interest to apply leading edge damage and observe the impact upon the vortex structure. In addition, progression to unsteady computations, supported by rig scale radial profile validation, is an important step to evaluate transient behavior in the damaged flow field and bulk rotational speed. In this instance a mixing plane would not be used and an accurate representation of the OGV blockage circumferential variation, downstream of a non-axisymmetric rotor damage profile, can be made.

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REFERENCES