

THE BOUNDARY LAYER SEPARATION FROM STREAMLINED SURFACES AND NEW WAYS OF ITS PREVENTION IN DIFFUSERS

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ABSTRACT

This work provides a detailed analysis of all force conditions acting in the limits of the boundary layer on the moving fluid and gas media, and shows that a value of the transverse gradient of a shear stress $\frac{\partial \tau}{\partial y}$, directly on the streamlined surface, is a main condition determining the possibility of attached flow along the streamlined surface in divergent zones.

From an analysis of all the forces and their development in the converging flow, an effective way of prevention of the flow separation from streamlined surfaces of the cone-type, ring-type and axial-radial diffusers was developed that provides the flow stabilization in the wide-angle diffusers and multiple reduction of the dynamic load level acting on the walls of the mentioned devices. Results of the CFD-modeling and experimental investigations for flat asymmetric and wide-angle conical diffusers are presented in this paper.

KEYWORDS

DIFFUSER, SHEAR STRESS, TRAVERSE GRADIENT OF SKIN FRICTION, FINNING, BOUNDARY LAYER SEPARATION

NOMENCLATURE

Latin Symbols

| | |
|-----------|---|
| \vec{c} | velocity vector |
| d | diameter |
| F_p | force due to longitudinal pressure gradient |
| F_τ | friction force |
| F_u | internal force |
| L | length |
| p | pressure |
| ρ | density |
| u, v, w | projections of the velocity vector to the coordinate axis |
| V | volume |
| x, y, z | coordinate system |

Greek Symbols

| | |
|----------|----------------------------|
| α | diffuser opening angle |
| μ | dynamic viscosity |
| τ | shear stress |
| τ_M | molecular friction tension |

| | |
|---------------|------------------------------|
| τ_T | turbulent shear stress |
| δ | boundary layer thickness |
| Abbreviations | |
| CFD | Computational Fluid Dynamics |

INTRODUCTION

The problem of flow separation from the smooth-contoured streamlined surfaces is one of the major problems among the challenges of modern fluid and gas dynamics, in both theory and practice, as when the separation occurs the flow pattern critically changes. This phenomenon leads to a drastic deterioration in equipment performance.

When the flow separation happens, the steady flow turns into unsteady. If motion of working fluid takes place inside the channels, during the flow separation from its walls, the hydraulic resistance increases dramatically, non-recoverable energy losses grow, acoustic vibrations accrue and the dynamic forces impacting on all surfaces located in the separation rise sharply.

Due to the mentioned negative effects associated with the flow separation from the streamlined surfaces in different channels, the problem of maintaining attached flow in the flow sections of industrial equipment becomes of all most importance. However, for resolving at least a part of this problem, it is necessary to have a clear idea of the mechanism of flow separation from the streamlined surfaces.

The study of flow in diffusers of different geometric characteristics, as well as the flow separation formation mechanisms, is devoted to a fairly wide number of scientific works. There is the article of Törnblom O. (2003), which provides extensive data of numerical and experimental studies of flow behavior in flat diffuser channels. Some characteristics of the diffusers with different opening angles are also presented (Chandavari V., Palekar M.S., 2014).

Conical diffusers were considered by Lenarcic M. et al. (2013). The authors do not only give the calculations results for different flow regimes, but also offer some optimization methods for diffuser and exit chamber design. The possible diffuser modernization was also studied by Singh S.N. et al. (2009), where a momentum injection by rotating cylinder was applied to control the boundary layer separation in S-shaped rectangular diffusers with different area aspect ratios.

Based on theoretical assumptions and hypotheses a method of controlling the occurrence of separation (fully or partially) by the installation of various configurations of fins has been developed and experimentally investigated. The results of symmetrical flat diffuser studies with longitudinal trapezoidal fins are given in an article of Zaryankin A.E. et al. (2012) and in book (Zaryankin A.E., 2014). Data on the annular finned diffuser for pre-swirling inflow are published in the work of Zaryankin A.E. et al. (2012).

In all cases, there is a presence of significant stabilizing influence of longitudinal finning of wide-angle diffusers both on flow separation zones' position and vibration conditions of the diffusers.

This article presents the results of the investigation of fin installation on the flow separation in different diffuser channels: in the flat asymmetrical diffusers and wide-angle conical diffusers (in the absence of swirling flow on diffuser inlet).

THE AERODYNAMIC INTERPRETATION OF FLOW SEPARATION PHENOMENON FROM THE SMOOTH STREAMLINED SURFACES

The existing view on the separation mechanism

The phenomenon of flow separation from a smooth streamlined surface in the majority of scientific papers is considered on a qualitative level with the involvement of simple mechanical analogies. Loitsyansky L.G. (1970) ties its appearance only to the amount of kinetic energy in the near-wall region, which should provide fluid movement against a pressure rise in the direction of its movement. Practically the same explanation of the flow separation reasons from the streamlined wall

is contained in the book of Schlichting H. (1969). The unsteady boundary layer separation considers in the articles of Telionis D.P. and Tsahalis D.T. (1974, 1976).

Classic interpretation of the boundary layer separation mechanism from the streamlined surfaces in the diffuser flow region is based essentially on the analogy of solid body that rises in the mountain. In this case, the inertial forces, the frictional forces and the force of gravity, forwarded in the opposite direction of movement, are elected on the moving body; any fixed position, the velocity of all points of the body remains uniform, and the friction forces act only on contact surface.

The situation changes significantly, when we consider moving liquids and gases. The velocity and the shear stress of friction change with the distance from a streamlined surface into the cross section of the boundary layer, and the nature of these changes is determined by the sign of the local longitudinal pressure gradient (Zaryankin A.E. et al, 2015, 2016).

Thus, existing view of the reasons for the boundary layer separation does not consider the pattern of change all of the existing factors acting within the boundary layer and determining the flow type in the diffuser channels.

The analysis of factors that determine flow nature within the boundary layer

Let us analyze the factors, which define fluid behavior within the boundary layer, on the basis of averaged Prandtl's equations for the two-dimensional flow:

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = - \frac{dp}{dx} + \frac{\partial \tau}{\partial y}, \quad (1)$$

$$\frac{\partial p}{\partial y} = 0 \quad (2)$$

where u and v – projection of a velocity vector \vec{v} into coordinate axes; $\tau = \tau_M + \tau_T$ – total shear stress within the boundary layer consisting of molecular shear stress $\tau_M = \mu \frac{\partial u}{\partial y}$ (μ – dynamic viscosity) and turbulent shear stress τ_T .

As defined, on the external border of the boundary layer $\left. \frac{\partial \tau}{\partial y} \right|_{y=\delta} = 0$. Thus, Eq. (1) goes over into

Euler equations for the two-dimensional flow of the ideal fluid.

Further, from Eq. (1) (on the streamlined surface) follows next:

$$\frac{dp}{dx} = \left. \frac{\partial \tau}{\partial y} \right|_{y=0} \quad (3)$$

In other words, a sign of longitudinal pressure gradient $\frac{dp}{dx}$ is the same as a sign of transverse gradient of the wall shear stress.

This condition is very important in the analysis of the changes that occur within the boundary layer, which develops in the diffuser channels.

At multiplication of Eq. (1) by the elementary fluid volume $dV = dx dy dz$, we obtain balance equation of all forces acting on the elementary fluid particle moving within the boundary layer:

$$dF_p = dF_\tau - dF_u \quad (4)$$

Here, external force $dF_p = \frac{dp}{dx} dx dy dz$ due to the longitudinal pressure gradient is balanced by

shear forces $dF_\tau = \frac{\partial \tau}{\partial y} dx dy dz$ and inertial forces $dF_u = \rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) dx dy dz$.

If the force dF_p , according to the second Prandtl's equation, does not change in the cross section of the boundary layer, then the values dF_τ and dF_u are always changing in such a way that their algebraic sum had remained constant in any cross section of the boundary layer and equal to the external force dF_p . Accordingly, the fluid motion along the streamlined surface is unseparated until the specified condition is fulfilled, i.e., until the diagram deformation of the force distribution dF_τ and dF_u under the influence of external (in this case geometric) forces provides compensation of the force dF_p .

Qualitative change of the distribution diagram of the shear stresses and transverse gradient for convergent $\left(\frac{dp}{dx} < 0 \right)$, gradientless $\left(\frac{dp}{dx} = 0 \right)$ and diffuser $\left(\frac{dp}{dx} > 0 \right)$ flows are shown in Fig. 1 and Fig. 2.

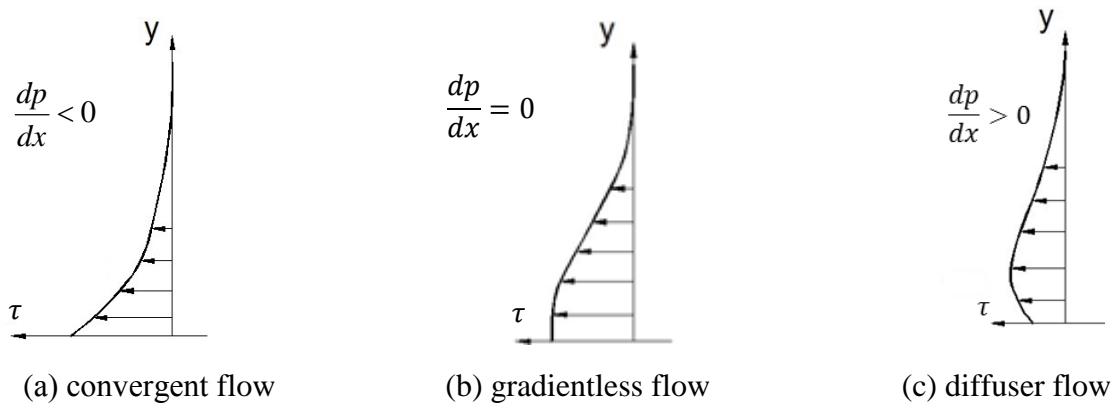


Figure 1: Diagrams of the shear stress distribution within boundary layer for different flow types

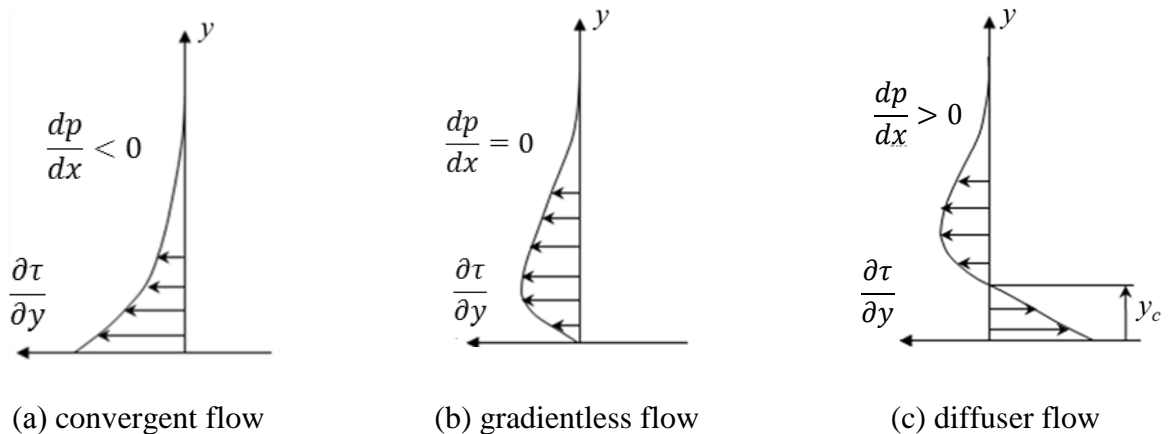


Figure 2: Diagrams of the transverse gradient of skin friction distribution within the boundary layer for different flow types

Thus, shear forces dF_τ in cross section of the boundary layer are qualitatively changing also.

The conclusion that the shear forces in the diffuser flow region at distance $y < y_c$ are directed in the same direction as the fluid flow is an essential result of the above analysis. In other words, at this distance the overlying layers drag the underlying (stopped) fluid layers, hence, providing the principal possibility of unseparated flow at a positive pressure gradient.

The fact that determines the possibility of movement of liquids and gases in the diffuser areas was first pointed out by Prandtl L.

If we add up the distribution of all forces acting within the diffuser boundary layer, then for the unseparated flow the balance of stress factors given by Eq. (4).

The distance from the wall y_c , at which a positive value of the transverse gradient of skin friction (shear force dF_τ) that makes possible flowing of the fluid in the near-wall region against the force dF_p acting in the opposite direction remains the same, depends directly on a value of the positive pressure gradient $\frac{dp}{dx}$. The greater the longitudinal pressure gradient $\frac{dp}{dx}$, the greater the value of the coordinate y_c . In other words, value y_c determines the response level of the moving fluid medium to the external (in this case geometric) influence determined by the pressure gradient $\frac{dp}{dx}$.

However, the possibility of a flow response to external influence has certain limits, which, if exceeded, causes flow separation from the streamlined surface.

In addition, the known conditions in the separating point of the boundary layer (shear stress on the wall ($\tau_w = 0$) and the transverse velocity gradient ($\frac{\partial u}{\partial y} = 0$) in separation point should be equal to zero), are to be supplemented with the condition of equality to zero on the wall of the transverse shear gradient ($\left. \frac{\partial \tau}{\partial y} \right|_{y=0} = 0$). The first two conditions can be referred to the necessary conditions for

the flow separation. The third condition ($\left. \frac{\partial \tau}{\partial y} \right|_{y=0} = 0$) is already a sufficient condition for the flow separation.

For a detailed review of the physical essence of these conditions, let us consider the shear stress distribution and the transverse gradient of skin friction distribution in the cross sections of the boundary layer located at different distances from the entrance of the diffuser channel. The diffuser with constant longitudinal pressure gradient ($\frac{dp}{dx} = const$) was selected for the analysis (Fig. 3).

The first section 1-1 is located near the entrance to the diffuser, the second section 2-2 – in the middle and the third section 3-3 – near the separation point of the boundary layer from the streamlined surface.

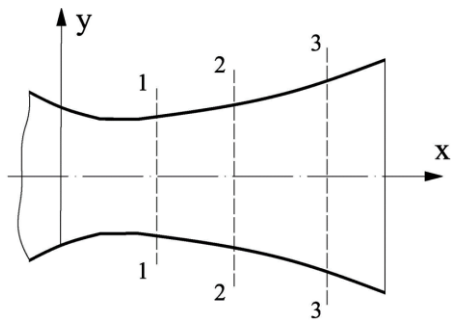


Figure 3: The cross sections location in the diffuser

The three boundary conditions were used to determine the diagrams of the shear stress distribution and the diagrams of the transverse gradient of skin friction distribution in the selected sections:

- in the absence of flow separation from the streamlined surface: $\frac{\partial \tau_w}{\partial y} \Big|_{y=0} = \frac{dp}{dx} = const$.
- at the outer edge of the boundary layer: $\tau_w \Big|_{y=\delta} = 0$, $\frac{\partial \tau_w}{\partial y} \Big|_{y=\delta} = 0$.

In addition, shear stress τ_w continuously decreases towards the outlet section of the diffuser. Respectively, boundary layer thickness δ grows.

The diagrams of the shear stress distribution and transverse gradient of skin friction distribution in the considered sections are shown in Fig. 4. The values of shear stress τ_w and transverse gradient of skin friction distribution $\frac{\partial \tau}{\partial y}$ are shown on the x-axis. Values of boundary layer thickness are shown by numbers on the y-axis.

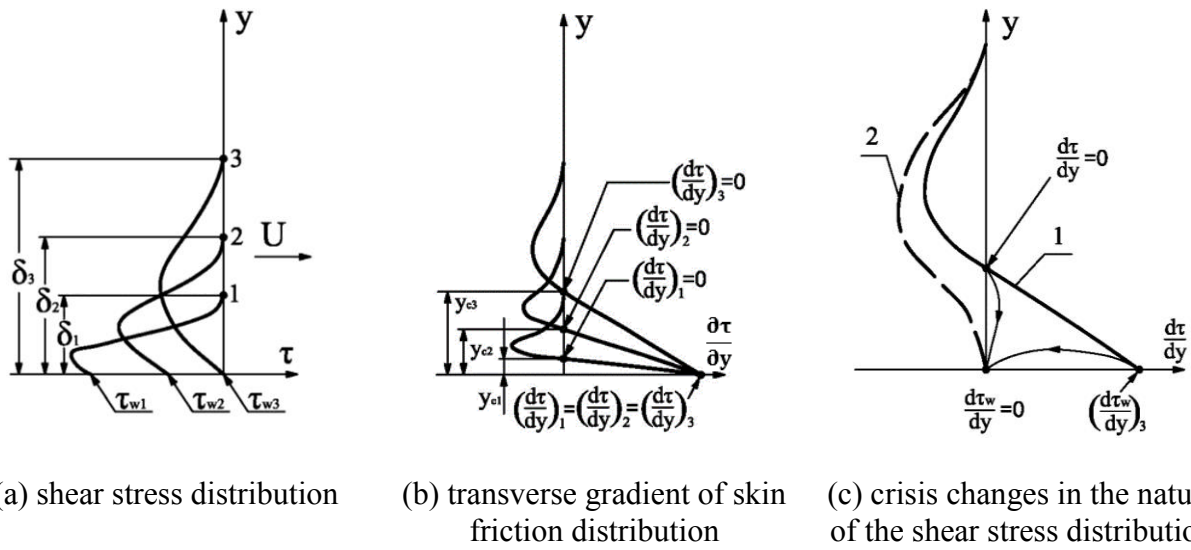


Figure 4: The qualitative picture of changes of the shear stress distribution and the transverse gradient of skin friction distribution in the cross section of the boundary layer

The curves of the shear stress distribution presented on the Fig. 4a show that the shear stress continuously decreases towards the outlet section of the diffuser until it becomes equal to zero ($\tau_w = 0$) on the wall in the cross section 3-3 (Fig. 3). According to the book of Loitsyansky L.G. (1970) and Schlichting H. (1969) the condition $\tau_w = 0$ defines the section location, where the flow separation occurs. However, from the Fig. 4 it follows that $\frac{\partial \tau}{\partial y} > 0$, when the shear stress is equal to zero. The transverse gradient of skin friction still balances the longitudinal pressure gradient $\frac{dp}{dx}$. In other words, the Eq. (4) still remains in force provided that $\tau_w = 0$. The possibility of longitudinal flow motion without separation from the streamlined surface when $\tau_w = 0$ has been demonstrated in the work of Ginevskiy A.S. (1968).

Thus, the condition $\tau_w = 0$ is the condition of unstable pre-separation state of boundary layer that determines the maximum possible response of the near-wall layer on positive pressure gradient.

Accordingly, any small perturbation that violates this delicate balance, leads to crisis changes in the nature of the shear stress distribution directly in the region of the solid streamlined surface (Fig. 4c).

The curve 1 shown in Fig. 4c corresponds to the distribution of the shear stress in a cross section of boundary layer in the pre-separation state ($\tau_w = 0$). The direction of the shear stress can be positive or negative (relative to the moving flow) under small perturbations. The value τ_{w3} decreases to zero and the shear stress distribution curve takes the form of the curve 2 (Fig. 4c). The point located at a distance y_{c3} from a solid wall shifts to this wall. Accordingly, when $\frac{\partial \tau}{\partial y} = 0$, further flow movement near the wall becomes impossible in the direction of the main flow. The flow separation from a streamlined surface occurs. The flow is unstable and longitudinal pressure gradient $\frac{dp}{dx}$ is close to zero after the flow separation point.

The process of transition from unseparated to separated flow is similar to the process of rupture of the wire rope during its continuous loading by external forces.

The gradual increase in the response of metal rope occurs under such loading in the form of increased tension until the rope breaks. The tension critically reduces to zero in the remaining part of the rope. This analogy gives reason to believe that the crisis reduction to zero of the transverse gradient of skin friction $\frac{\partial \tau_w}{\partial y}$ is a sufficient condition for the flow separation from the streamlined surfaces.

Methods of prevention of flow separation in the diffuser channels

The physical picture of the flow separation appearance from an aerodynamic surface and a mathematical formulation of its inevitable appearance conditions $\left(\frac{\partial \tau}{\partial y} \Big|_{y=0} = 0 \right)$ allows to develop new effective methods of influence on the flow nature in various types of diffusers.

As mentioned before, the degree of flow response to the external geometrical effects is determined by the value $\frac{dp}{dx}$. Within the boundary layer, where flow occurs with a large transverse gradient of the longitudinal velocity u , the flow response is accompanied by a change in the inertial forces, friction forces τ and the variation of the transverse gradient $\frac{\partial \tau}{\partial y}$.

The value $\frac{\partial \tau}{\partial y} \Big|_{y=0}$ is the only type of flow response to an external geometric effect at the surface due to no-slip condition.

Accordingly, the wider the range of possible changes of the shear stress and the transverse gradient of skin friction, the more the maximum value of the longitudinal positive pressure gradient, at which a non-separable flow can be implemented in the diffuser channel.

In particular, the shear stress can be substantially increased due to special longitudinal finning of the diffuser channels, because the implementation of fins greatly increases the total frictional force at the streamlined surfaces. The negative values τ_{w1} , τ_{w2} shown on the Fig. 4a have increased after the longitudinal finning of the diffuser. The transition to pre-separation velocity profile (Fig. 4a) occurs at a much greater distance from the diffuser inlet section after the longitudinal finning.

NUMERICAL AND EXPERIMENTAL INVESTIGATION OF THE FINNING DIFFUSERS

Numerical study

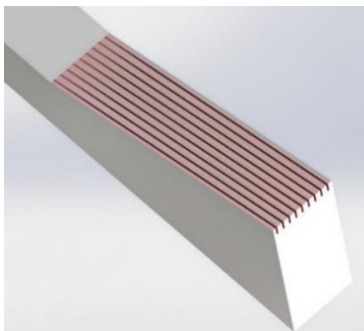
The investigation of the effectiveness of longitudinal fining for improving the flow characteristics in the diffuser channels has been made for flat and wide-angle conical diffusers.

Flat asymmetrical diffusers with different opening angles ($\alpha = 5, 7.5, 8.5, 10, 15^\circ$) and the various design options and layouts of the fins was studied numerically by mathematical modeling methods. The example of flat diffuser with wedge-shaped fins of rectangular cross section is shown in Fig. 5.a. Maximum height of fin is equal to 8 mm, width – 1.5 mm and length – 270 mm. Fin pitch is equal to 5 mm.

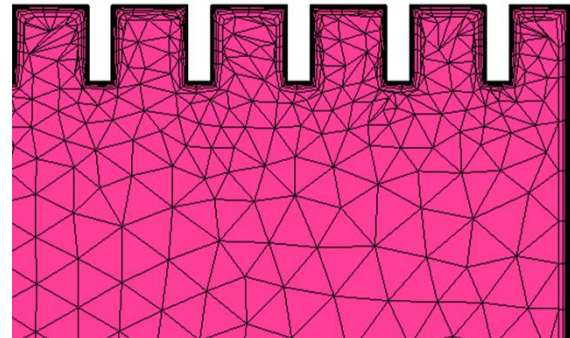
The computational mesh of the considered diffusers have a size equal to the average of 2-3 million cells (Fig. 5b); mesh is unstructured; the boundary layer composed of prismatic elements (the number of prismatic layers: 13-15). Suggested flow separation area, finned surface and the space between them were resolved in more detail for obtaining a stable and accurate solution.

The flow simulation in a flat diffuser was carried out using the software package ANSYS CFX. As boundary conditions at diffuser inlet was applied developed velocity profile (obtained by solving the problem of fluid flow in a long channel of rectangular section with ratio $L/d > 50$); the static pressure was set as output condition. Wall of the model was defined variously depending on their geometric location.

For this study, the turbulence model $k-\omega$ gave the highest qualitative convergence in the modeling of flow separation in diffusers among all two-parameter functions available software package ANSYS CFX. The problem was solved as steady-state.



(a) model



(b) computational mesh

Figure 5: Model and computational mesh of flat finned diffuser ($\alpha = 15^\circ$, expansion ratio = 2)

As shown by the results of the numerical experiment (Fig. 6), the longitudinal fins really helps to stabilize the flow structure.

Numerical study was also carried out for wide-angle conical diffusers. An example of one of the investigated models is shown in Fig. 7a. In this case, wedge-shaped fins of rectangular cross section were also applied. Maximum height of fin is equal to 2 mm, width – 0.5 mm and length – 170 mm. Fin pitch is equal to 5 mm.

The size of meshes was 2-3 million cells, the number of prismatic layers: from 13 to 15 (Fig. 7b). The mesh resolution of the flow inside the diffuser near the finned walls were made more detailed than the mainstream flow volume. The mesh parameters allowed to obtain an accurate and stable solution.

Non-uniform velocity profile, derived by simulation of fluid flow in long cylindrical channel, was applied as boundary conditions at the diffuser inlet. At the outlet static pressure. The wall was modeled smooth, adiabatic and with no-slip condition.

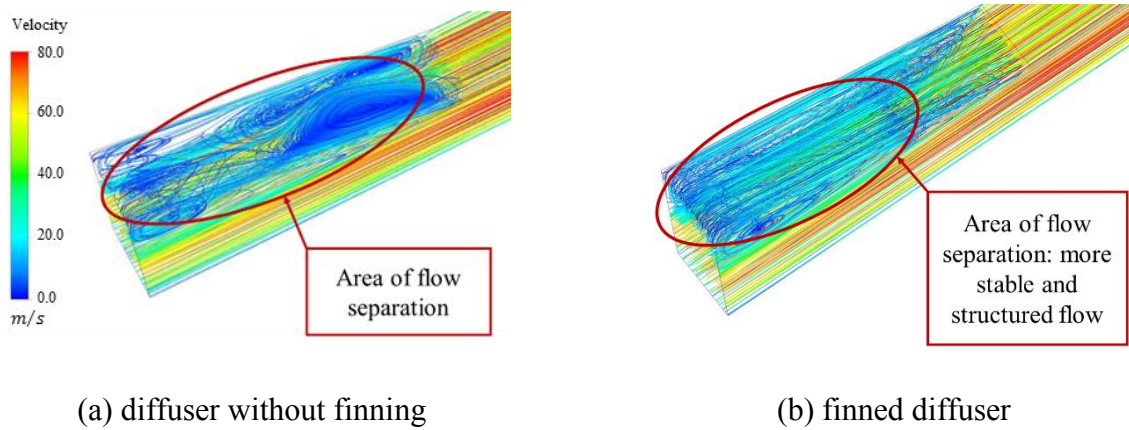


Figure 6: Streamlines in the diffuser ($\alpha = 15^\circ$) according to the results of flow simulation

The turbulence model used – $k-\omega$. The problem was solved in a stationary and unsteady conditions using the software package ANSYS CFX.

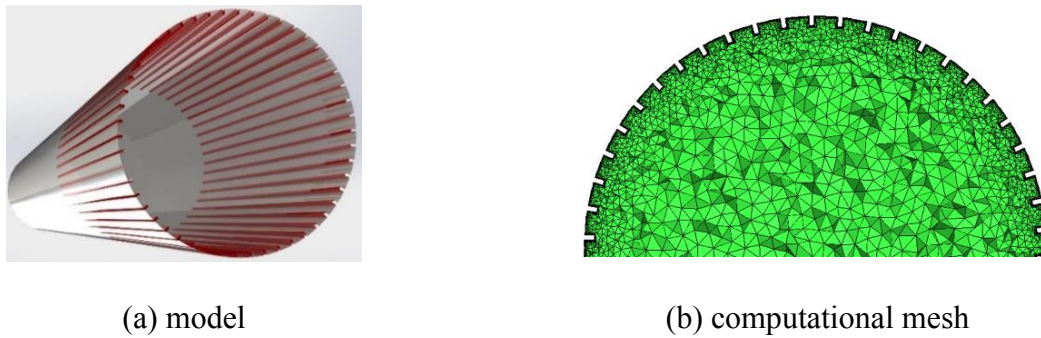


Figure 7: Model and computational mesh of conical finned diffuser ($\alpha = 15^\circ$, expansion ratio = 10)

The fins installation efficiency in the conical angle of diffuser channels resulted in the improvement of diffuser performance: more stable and structured flow. The modeling results for the conical diffuser with $\alpha = 15^\circ$ are given in Fig. 8.

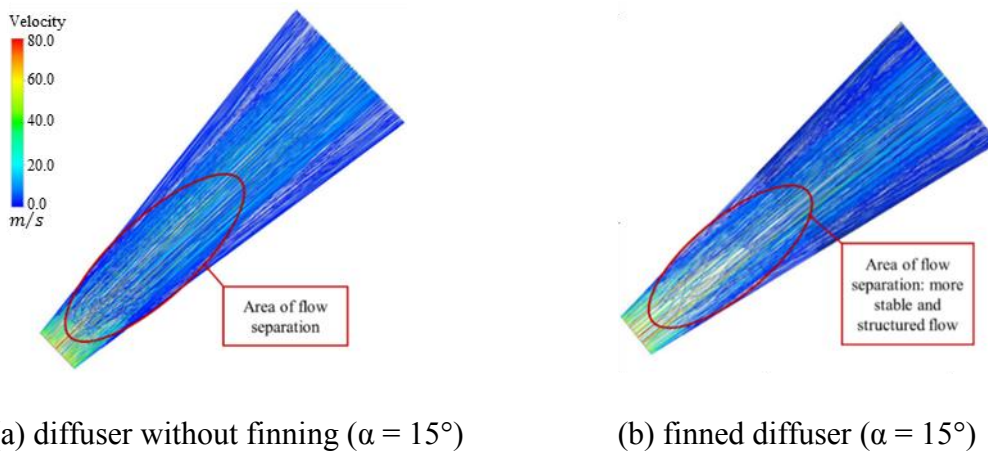


Figure 8: Streamlines in the diffuser according to the results of flow simulation

The quantitative evaluation of the finning method effectiveness is performed by determination of the pressure recovery coefficient. For diffusers with small opening angle ($\alpha < 15^\circ$) installation of the

fins contributes to higher losses of energy flow. In the absence of flow separation, the finning leads only to an increase of the friction loss.

The pressure recovery coefficient was increased from 0.31 to 0.35 for wide-angle flat diffuser ($\alpha = 15^\circ$, expansion ratio = 2) and from 0.4898 to 0.4914 for wide-angle conical diffusers ($\alpha = 15^\circ$, expansion ratio = 10). The improvement of diffuser efficiency in both case is not so high. Nevertheless, this fact demonstrates a positive effect of the finning method on flow despite the fact, that they contribute additional aerodynamic drag.

Experimental study

An experimental study was conducted for the flat diffusers for baseline (without finning) model and model with longitudinal fins. The experimental unit used in the experiments is shown in Fig. 9.

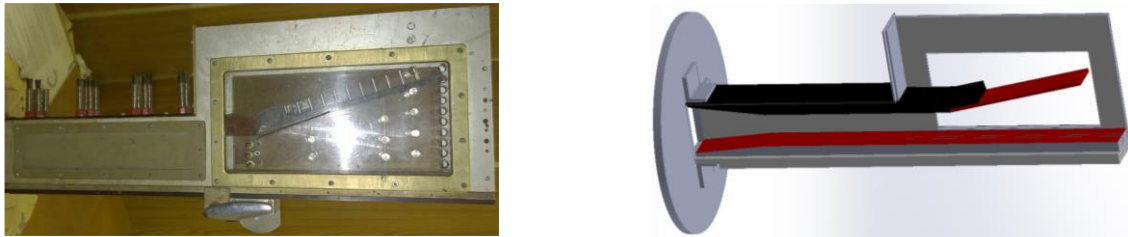


Figure 9: The experimental unit for investigation of flat steel diffuser

In the series of experimental tests spectrograms of the pressure pulsations on the walls of the flat diffuser with opening angle $\alpha = 15^\circ$ (Fig. 10), in different sections during the flow in the absence of the fins (a) and with the installation of the wedge-shaped fins (b) were obtained.

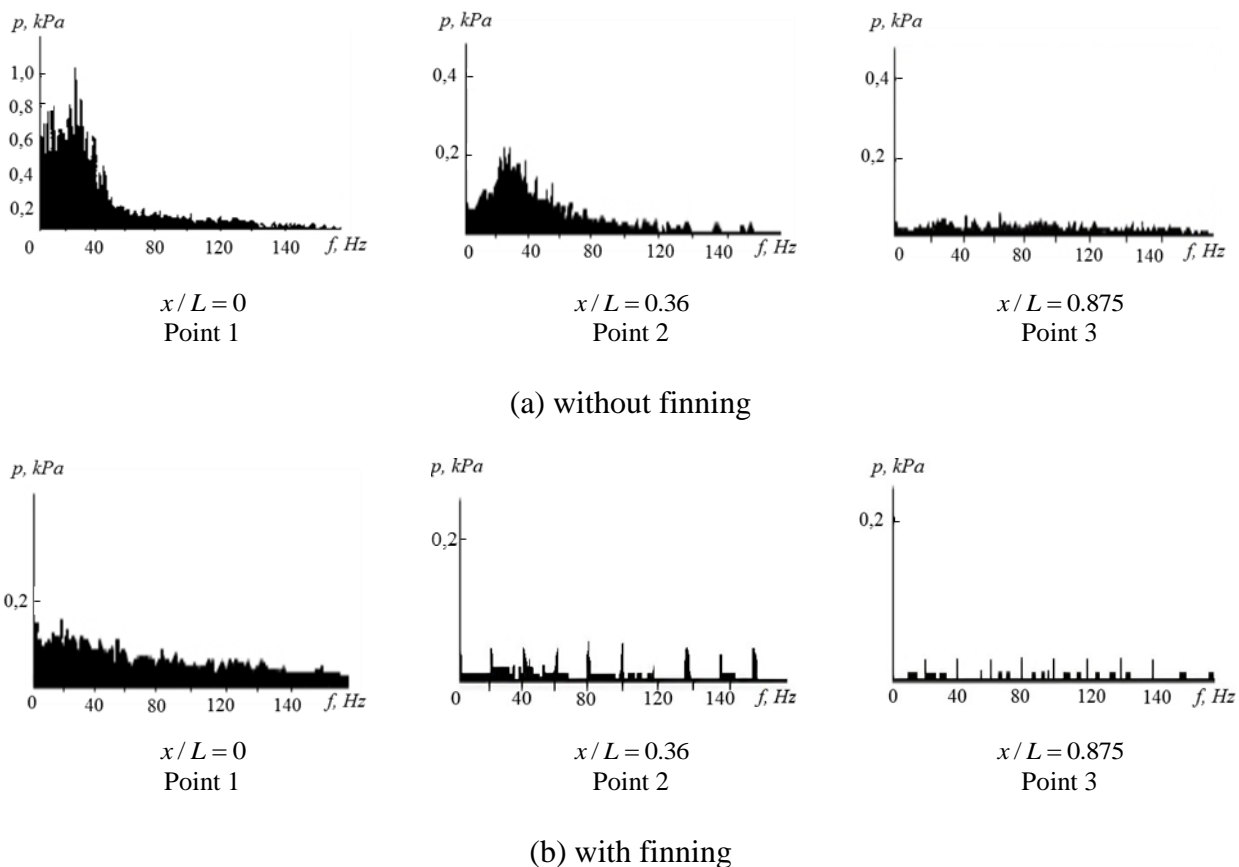


Figure 10: The spectrograms of pressure fluctuations on the diffuser wall along the length of the flat diffuser channel ($\alpha = 15^\circ$)

A comparison of these spectrograms shows multiple reducing of pressure pulsation amplitudes obtained by fins application, moreover the pulsations' amplitudes decrease sharply in the low frequency range.

This fact improves significantly the vibrational state of diffuser, and the opening angle $\alpha > 15^\circ$ also provides some reduction in energy losses along the length of the diffuser channel.

CONCLUSIONS

1. It is shown that existing scientific concepts do not describe properly the mechanism of boundary layer separation from the streamlined surfaces and accordingly do not allow to develop effective methods of avoiding separation.

2. It is revealed that the separation of the flow from the smooth streamlined surfaces is a critical flow state, at which the moving fluid possibilities to respond to the inertial forces and friction forces are completely exhausted. Therefore, for the attached flows it is necessary to affect the flow behavior and the geometrical parameters of channel in such a way to provide significant response of the flow to external influences.

3. The boundary of flow separation will be determined by the value of the transverse gradient of skin friction for the diffuser channels. The lateral shear stress gradient of friction should be increased to obtain attached flows. It is proposed to use special longitudinal finning of the streamlined surfaces of the diffuser channels.

4. Numerical and experimental flow investigation the diffuser channels with mounted longitudinal fins showed the effectiveness of finning usage to improve the flow characteristics in the diffusers. In all considered cases, there was a noticeable manifestation of the stabilizing influence of the fins on the position of the separation region, the increase by the structuring of the flow, the reduction of pressure pulsations on the channel walls, and the reduction of diffuser vibration condition.

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