Transient flow in infinitely thin airfoil cascade

F. Hermet - N. Binder - J. Gressier

ISAE-SUPAERO, Université de Toulouse, France
florian.hermet@isae-supaero.fr

ABSTRACT
Pulsed flows in turbines have many industrial applications. However, the influence of pulsed flow on turbine performance is complex and imperfectly understood. There is no clear consensus in the literature regarding generic trends. Those are difficult to extract since a majority of studies are based on industrial geometries, which involve strong interactions between many physical phenomena. In order to isolate and understand the main expression of the physics, the problem is simplified and an investigation is carried out on the severe transient regime inside a thin airfoil cascade, caused by a sudden change of inlet conditions, without viscosity effect. Different time scales of the inlet perturbation are examined, from a sudden step of the flow features to a more progressive ramp. The consequences of the different regimes observed inside the flow regarding some indicators of performance are detailed. Finally, the characteristic time of the transient flow is isolated and proposed as an indicator to assess the relevance of the quasi-steady assumption.

KEYWORDS
Pulse, step, ramp, overflow, overload, time scale

NOMENCLATURE

\( \beta \) Guide vane angle
\( t \) Time
\( t^* \) Normalized time by \( t_{ua} \)
\( \Delta t^* \) Non-dimensional number dictating the unsteady response
\( \Delta P \) Pulse pressure variation
\( t_{ua} \) Shock propagation time through the geometry
\( F \) Aerodynamic effort
\( l^* \) Axial length of the geometry
\( L \) Lift-to-drag ratio
\( \dot{m} \) Mass flow rate
\( n \) Blade normal vector
\( p \) Static pressure
\( P_i \) Stagnation pressure
\( T_i \) Stagnation temperature

Subscripts

0 Initial state
\( f \) Final state
\( RH \) Downstream state at initial shock wave

INTRODUCTION
In many applications, the turbine is subject to temporal variations of its inlet conditions. The most extreme cases are found when the turbine is located just downstream of an isochoric combustion chamber. And the possible evolution of aeronautical engines from isobaric toward isochoric combustion is currently under careful examination since the thermal efficiency of the cycle is reputed higher. However major contributions on this question are credited to the automotive turbocharger community. In Baines (2010), there is a detailed summary of nearly 20
years of research on pulsed flow in turbines. This review highlights the progress and contradictions in the scientific community, particularly regarding the pulsed flow influence on turbine performance. These contradictions are mainly related to the difficulty of finding a consistent pulse characteristic time scale: the pulse effect is not only characterized by the flow advection but also by its acoustic propagation. Thus, a coherent characteristic time scale of the pulse effects cannot only be determined using the pulsed regime frequency. The comparison between this pulsed regime frequency and the rotor frequency suggests a possible quasi-steady behaviour, but this conclusion is less obvious if the comparison is performed with the flow based propagation time-scale of the pulse. This is a matter of importance since an easy 1D modeling of the turbine is practicable under the quasi-static assumption of the rotor. And effectively, there is a general trend in the community to consider that the non quasi-steady behaviour is caused by the stator, for which the fillings/emptyings of the stator volume induce some phase shift. The rotor is treated as a quasi-steady device, even though this needs to be more precisely assessed.

Several criteria have emerged to determine whether the flow can be considered as quasi-steady or not. The most famous is based on the Strouhal number, but more recent criteria based on the temporal pressure gradient have emerged (Copeland et al. 2012, Cao et al. 2013). Cao et al. (2013) proposes a local criterion based on the time-derivative of the inlet pressure which is normalized and for which a threshold value is proposed, above which the unsteady forcing gives rise to a non quasi-steady behaviour of the rotor. In this criterion, the pressure is normalized by an average pressure while the time is normalized by an advective time: the time for a fluid particle traveling through the turbine passage.

Nowadays, flow unsteadiness is considered damaging for turbine performance. However, Gugau & Roclawski (2012) suggest that an over-torque can be recovered by the rotor, while Binder et al. (2013) notice that the rotor response is unusually high for some opening of guide vane. However, no explanation is given on the nature of the physical phenomenon.

As aforementioned, most of the studies of the literature, experimental or numerical, were performed on complex industrial configurations where multiple physical phenomena are coupled. This partly explains why there is no consensus on the pulsed flow influence in turbines. Since a single pulse is mainly characterized by its advection and propagation, this paper focuses only on the inviscid effects of a single pulse in a thin airfoil cascade, with the target to clarify the transient regime physics and catch first trends of the unsteady performance. The first part of this paper presents the numerical simulation set up to carry out the study. The second part focuses on the transient flow analysis, during a sudden regime change which provokes a shock wave propagation. The analysis of a more progressive variation of the admission conditions is also considered. In these two studies, the stagnation temperature is kept constant through the single pulse, whereas the last part is devoted to the influence of the stagnation temperature variation upstream of the turbine.

**CONFIGURATION DESCRIPTION**

The cascade (Fig. [1]) used for numerical simulations is two-dimensional; the blades are skeletal in order to remove unwanted thickness effects such as wave reflections and reduce number of parameters in the problem.

The axial length of the computed domain is equal to $l^*$. The chord of the blades is $0.2l^*$, and are separated by $0.04l^*$. The throat is located at the trailing edge of blades ($0.4l^*$). The guide vane angle at trailing edge is $\beta = 50^\circ$. This configuration thus has the main features of a stator channel or the channel of a rotor attached to the relative frame.
An inviscid approach is chosen to focus the analysis on advective and propagative effects. So the investigations are performed by solving the Euler equations. Numerical simulations are carried out with the solver: CharLES\textsuperscript{X} (Bermejo-Moreno et al. 2014).

The mesh contains $500 \times 100$ quads in respective axial and tangential directions. Boundary conditions are defined as described in Fig. [1]: wall boundary conditions are also prescribed upstream the expected leading edge of the guide vane since a periodicity condition in this region would only have added unwanted disturbances from the leading edge which have very little interest for the mean flow study.

In the literature, a pulsing flow is generated by imposing a time periodic discontinuity or a periodic progressive variation at the turbine inlet. The present work only focuses on unique pulse influence (i.e a step or a ramp) generated by the inlet boundary condition (Fig. [2]).

For the case of a step variation of the inlet boundary condition, two cases exist. Either the discontinuity imposed in the boundary conditions is solution of the Rankine-Hugoniot equations and the pulse consists in a single shock wave, or the discontinuity is generic, and the pulse is constituted by a shock wave followed by a contact surface. For a ramp, the pulse is represented by a compression wave if the variation satisfies the equations of an unsteady isentropic wave, or else, the pulse is formed by a compression (and focusing) wave and a varying entropy region which respectively travel at acoustic and flow speeds.
In the two following sections, the inlet boundary conditions are modified in such a way the inlet stagnation temperature is kept constant through the single pulse. The influence of stagnation temperature variation is discussed in the last part (Fig. [2]). The transients imposed to the boundary conditions in this paper are generic, and do not satisfy the Rankine-Hugoniot jump equations. Thus, an entropy wave is also necessarily created at the boundary condition.

**SINGLE PULSE GENERATED BY A STEP OF STAGNATION PRESSURE**

Several types of transient regime exist according to the pulse intensity and the final state of the flow (subsonic or supersonic). In this study, the transient flow examination is restricted to the cases with a subsonic initial state and a supersonic final state downstream to the throat. Thus, results and trends on the steady performance indicator of the turbine which will be further explained are limited to this range of initial and final parameters.

A brief description of the transient regime physics is provided in order to better understand the temporal evolution of the mass flow and aerodynamic force.

**Brief description of the transient flow**

The \((x, t)\) diagram is used to describe transient regime physics, as usually found when dealing with unsteady flows. The \((x, t)\) diagram provides a 1D representation of the flow and highlights the waves propagation during the transient flow. In the present study, as most phenomena are mainly 1D, we choose to extract a 1D representation of the flow along the mean line of guide vane. Potential 2D effects, if acting in the flow, will be additionally discussed.

The \((x, t)\) diagram of Mach number is represented in Fig. [3]. All propagating waves are visible on the Mach number diagram since the Mach number changes through a shock wave, a contact surface, a compression wave or an expansion wave.

![Mach Number Diagram](image)

**Figure 3:** \((x, t)\) diagram of the Mach number for a discontinuous pulse (left). 2D visualization of the Mach number in final steady regime (right).

The figure [3] depicts the propagation of the shock wave and contact surface generated by the total pressure discontinuous jump on the inlet boundary condition at \(t^* = 0\).
The shock wave diffracts in the cascade due to the guide vane deviation which acts as a decrease of the cross-section. A diffraction appears according to the theory developed by Chester (1954), Chisnell (1957) and Whitham (1958). This diffraction induces a transmitted shock wave and a contact surface, as well as a reflected compression wave (see Whitham (1958) for more information). The reflected wave then interacts with the inlet boundary condition, producing a streamwise expansion wave, which will also diffract at the throat. The characteristic time of the first interaction between the compression wave and the inlet boundary condition is named $t_{u-a}$. While the time for which the shock wave interacts with the outlet boundary is named $t_{u+a}$.

This generic wave pattern is always observed, regardless of the initial, final flow regime and step intensity.

In the particular case shown in Fig [3], downstream of the throat, the shock induced acceleration makes the flow supersonic. This results in an oblique shock wave and a Prandtl-Meyer expansion at the trailing edge of blades which are revealed as a steady pattern in $(x,t)$ diagram.

**Analysis of mass flow rate and aerodynamic force**

Mass flow and aerodynamic loads are consistent quantities in the steady-state regime, which can be used to characterize the power extracted by the turbine. Therefore, they are expected to give an indication of the turbine performance during the transient regime even though the mass flow rate is not exactly representative of instantaneous performance.

The performance quantities on a stator can be analyzed as it would be on a rotor. Indeed, for a degree of reaction $\sigma = 50\%$, stator and rotor are symmetric. Moreover, it can be verified that the wavefront distortion is the same at the inlet of the stator, or at the inlet of the rotor (Fig. [4]). Thus, it can be assumed that the behaviour of the stator and the rotor channel attached to the relative frame of reference are similar.

![Figure 4: The standard deviation of the static pressure computed along a line normal to the mean-line at the detected shock wave position (left). 2D visualization of the shock wave at two different locations (right).](image)

In order to assess the shock wave distortion, the standard deviation of the pressure is com-
puted on a line normal to the mean-line at the detected shock wave position. The distortion and
the inclination of the wavefront are zero if the standard deviation is found equal to zero.

Thus, considering a distance between the rotor and the stator equal to the stator chord or
greater, wavefront distortion at the inlet of the rotor and stator are similar (Fig. [4]), which
supports the extension of our conclusions to the case of the rotor.

The indication regarding the mass-flow and the aerodynamic forces could thus be trans-
ported in the relative frame of the rotor, and then being representative of energy transfer.

**Mass flow rate**

The first phase of the transient regime is characterized by an intense overflow at the inlet
(Fig. [5]) compared to the final state. This overflow is caused and enclosed by the shock wave
followed by the contact surface which propagates at different speeds.

Shock wave moving in the flow direction is actually associated with a mass flow increase.
The contact surface also induces a mass flow rate elevation as the density increases as it passes
through. This can be explained by a decrease of stagnation temperature across the contact
surface to ensure a constant stagnation temperature through the pulse.

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![Figure 5: Temporal evolution of the mass flow rate at several sections for a step of \( P_{\text{inlet}} \).](image)

The overflow state persists until the reflected compression wave interacts with the inlet
boundary condition: this compression wave reduces the Mach number. The last oscillations are
due to multiple and decreasing intensity diffracted and reflected waves.

When \( T_{\text{inlet}} = \text{Cst} \) and the shock propagation chokes the guide vane, the overflow is always
present, since the wave system has the same pattern (backward reflection which decreases the
flow velocity, and interacts with the boundary condition, ...), which necessarily converges to-
w ard a lower mass flow than the post-pulse mass flow. In a subsonic regime, the response of
outlet boundary condition can modify this result.

Whichever the regime, the overflow at the throat is less intense than the one recorded at the
inlet. This is due to the modification of the state downstream the shock wave, since it diffracts
in the first part of the geometry (convergent geometry).

Another point is the variation of the phase shift between the inlet and the throat over time because the contact surface or simple waves do not move at the same speed as the shock wave. Moreover, this phase shift changes sign since the reflected compression wave is created at the throat and then interacts with the inlet. The phase shift is then influenced by both propagative (shock wave, simple waves) and convective (contact surface) effects.

Then it is necessary to try to maximize the phase related to the characteristic time $t_{u-a}$, to accentuate power recovered by the turbine. This phase actually induces a benefit on the mass flow rate compared to a steady flow.

**Aerodynamic force**

In Euler equation’s case, the aerodynamic force is only produced by the pressure distribution, which can be integrated, and decomposed in the usual Lift and Drag components:

$$ F = \int \int_{Blades\ surface} p \, u \, dS = Drag \cdot x + Lift \cdot y \quad (1) $$

The pressure distribution is unsteady and produces some overload (Fig. [6]) during the first phase of the transient regime. This overload is due to the shock wave diffraction, which produces some instantaneous values of the pressure, just downstream of the compression wave, which are much higher than the final steady-state regime.

Conversely, the expansion wave generated by the reflection of the compression wave with the inlet boundary as well as its diffraction at throat decreases the force before it converges towards the steady-state value. But this under-load does not match the overload and the average force applied during the transient is still higher than the final steady-state value.

The evolution of the lift-to-drag ratio is more complex to analyze, and its time-averaged value during the transient gets lower than the final value. But compared to a steady-state tran-
sient, a benefit is still observed. Thus, the brutal step of stagnation pressure seems to produce a significant benefit compared to the quasi-steady evolution.

**RAMP OF STAGNATION PRESSURE**

The previous section has shown that the temporal evolution of the mass flow and the aerodynamic force are dominated by propagative effects. When considering a pressure ramp, the time scale associated to the pressure time-evolution at the inlet becomes a parameter of the problem. From a physical point of view, the main consequence in the pattern is the replacement of the propagating shock wave and contact surface by a compression wave and an entropy gradient, which also diffracts inside the guide vane. A sensibility study of this time scale, regarding the former observations is now proposed.

**Influence of the temporal pressure gradient on mass flow rate**

The additional time-scale of the problem requires a reference. The time $t_{u-a}$ is selected and represents the time taken by the reflected compression wave (induced by the diffraction of the initial compression wave in the guide vane) to reach the inlet boundary condition ($t_{u-a}$ is defined in Fig. [3]). A non-dimensional number based on this characteristic time can be deduced:

\[ \Delta t^* = \frac{\Delta t_{ramp}}{t_{u-a}} \]  

(2)

Simply speaking, $\Delta t^*$ sets the ramp portion that is completed before the geometry response. When $\Delta t^* < 1$, the inlet variation is completed before that the geometry response interacts with the inlet boundary. In this case, the behaviour of the inlet mass flow temporal evolution is similar to that of a step (Fig. [7]).

![Figure 7: Temporal evolution of the inlet mass flow rate for several $\Delta t^*$ at $\Delta P^* (= \frac{\Delta P_{ramp}}{P_0})$ constant.](image)

When $\Delta t^* > 1$, the inlet variation is slow enough for the reflection to reach the inlet section
before the ramp is completed. In this case, the temporal evolution is more progressive and linear (Fig. [7]). An overflow is still present but less pronounced due to the geometry response.

The pulse amplitude may have some influence also on the transient flow, and this point is also investigated. A mapping is carried out by running many simulations in which the inlet normalized overflow is recorded for several $\Delta P^*$ and $\Delta t^*$ (Fig [8]).

The map shows a different behaviour for $\Delta t^*$ and $\Delta P^*$: when a brutal ramp is injected ($\Delta t^* < 1$), $\Delta P^*$ has a significant influence on the overflow. The normalized overflow increases when $\Delta P^*$ increases to a certain threshold, then it decreases. This trend can be illustrated by solving the equivalent Riemann problem, but this is not detailed in this paper. Moreover, the higher $\Delta P^*$ is, the longer the overflow phase lasts. Indeed, when $\Delta P^*$ is high, the reflected compression wave returns at a relatively low speed.

Figure 8: Map of percentage of inlet overflow as a function of $\Delta P^*$ and $\Delta t^*$. $\Delta P^* = \Delta P_{\text{ramp}} / P_0$. Overflow computed in comparison with the final steady state.

In the linear case ($\Delta t^* > 1$), $\Delta P^*$ does not influence the overflow anymore. It is only controlled by the value of $\Delta t^*$. The geometry response seems to eliminate the dependence of $\Delta P^*$ from the normalized mass flow. Therefore, with the current normalization of the mass flow, which focuses on the transient, regardless of the initial and final states, the value of $\Delta t^*$ imposes the overflow intensity.

However an implicit influence of $\Delta P$ still exists since it modifies $t_{u-a}$: the reflected compression wave velocity depends on the initial compression wave intensity. The conclusions about the mass flow can also be transposed to the aerodynamic effort study.

Thus, $\Delta t^*$ is the good indicator of the quasi-steady nature of the system response to the inlet pulsating flow. This criterion is not in contradiction with the one developed by Cao et al. (2013) but it describes at a finer scale the causes of the unsteadiness in the system response.
INFLUENCE OF DISCONTINUOUS VARIATION OF STAGNATION TEMPERATURE THROUGH THE SINGLE PULSE

The influence of an inlet step of stagnation pressure is now combined with an associated stagnation temperature variation. There are two different cases:

- **Case 1**: \( T_{\text{inlet}} < T_{\text{RH}} \)
- **Case 2**: \( T_{\text{inlet}} > T_{\text{RH}} \)

The previous study of an inlet step belongs to case 1. The main difference between cases 1 and 2 concerns the downstream state of the contact surface generated by the inlet boundary. In case 1, this state has a higher density than the upstream state, as explained in the devoted section. While in case 2, the downstream state has a smaller density due to the stagnation temperature imposed by the inlet boundary.

Therefore, the mass flow rate decreases through the contact surface. Thus, the overflow is less intense when \( T_{\text{inlet}} > T_{\text{RH}} \). The higher \( T_{\text{inlet}} \) is, the lower the downstream mass flow at the contact surface is.

In the case shown in Fig. [9], only the shock wave produces a real benefit on the mass flow compared to the final state. However, in the general case, an overflow may occur downstream of the contact surface (depending on the value of \( T_{\text{inlet}} \)) even if it will be lower than the post-shock mass flow.

Actually, the final mass flow depends on the value of \( T_{\text{inlet}} \), it decreases when \( T_{\text{inlet}} \) increases. This explains the difference of normalization on the mass flow rate (Fig. [9]).

![Figure 9: Temporal evolution of mass flow rate for \( T_{\text{inlet}} > T_{\text{RH}} \).](image)

The increase of \( T_{\text{inlet}} \) makes the initial shock wave more severe, thus the various waves of the transient regime are also more intense. Therefore, the instantaneous overload (Fig. [10]) is more pronounced and the oscillations in terms of magnitude are more significant. Here too, the overload is due to the shock wave diffraction.
The phase shift between the two cases changes over time since the wave-speed propagation is not the same. The average amplitude of the effort during the transient flow is equivalent for the two cases, contrary to the lift-to-drag ratio which is less favourable when $T_{\text{inlet}}$ increases.

CONCLUSIONS

In this paper, a transient flow description in thin-airfoil cascade has been detailed. Waves involved during the transient regime caused by a single pulse were described and their influences on the instantaneous mass flow and aerodynamic effort were analyzed to illustrate the prospect of power recovery. This leads to the conclusion that an unsteady forcing can produce an overflow and an overload (even though the lift-to-drag ratio is not optimized) compared to the final steady regime. Overflow and overload are present for a time equal to $t_{u-a}$ in the case of an inlet step. Consequently, the unsteady work recovered by the turbine should be more significant. The study of the inlet time-evolution of the pressure has highlighted a characteristic time in relation to the propagative effects which control the transient regime behaviour. Moreover, it has been found that the influence of the temporal pressure gradient on the overflow, in the case of a not abrupt ramp ($\Delta t^* > 1$), is expressed only through a non-dimensional number based on this time scale. The lower this number is, the greater the overflow is. Taking into account the stagnation temperature variation during the pulse attenuates the benefits in terms of overflow. However, an overload can always be recovered. During a pulsed and cyclic flow, these advantages must be cancelled during the downward phase, so it is imperative to optimize the pulsed flow cycle, in order to fully benefit from the advantages of an unsteady forcing.

Conclusions drawn on the aerodynamic force are only a first indication of those undergone on the rotor blades and should be refined by performing numerical simulation of a complete turbine where viscous effects and boundary layer interacting will be analyzed.
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