SPECTRAL PROPER ORTHOGONAL DECOMPOSITION OF COMPRESSOR TIP LEAKAGE FLOW

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ABSTRACT
Spectral proper orthogonal decomposition (SPOD) is performed on the near-stall tip leakage flow of a low-speed compressor rotor. The data used for the SPOD analysis is obtained by delayed-detached eddy simulation (DDES), which is validated against experimental data. The flow quantities of interest include the near-tip axial velocity and the blade surface pressure. Results show that the near-stall flow field of the investigated rotor is governed by two tip leakage vortices (TLV). The main TLV initiated from the leading edge exerts an unsteady force on the blade pressure surface. Its modal component is dominated by the leading modes at low frequencies. The secondary TLV originated from the mid-chord creates a weaker unsteady force on the blade suction surface, and its modal component has more high-frequency components due to its interaction with the suction surface boundary layer flows. These findings improve the understanding of the effects of tip leakage flow on compressor aerodynamic and aeroelastic stability.

KEYWORDS
SPOD, TIP LEAKAGE FLOW, DDES, AXIAL COMPRESSOR

NOMENCLATURE
BPFblade passing frequency
DDES delayed-detached eddy simulation
SPIV stereoscopic particle image velocimetry
SPOD spectral proper orthogonal decomposition
TLV tip leakage vortex
c\textsubscript{t}blade tip chord length
u\textsubscript{1}inlet axial velocity
U\textsubscript{t}blade tip speed
\rho\textsubscript{1}inlet density

1 INTRODUCTION
Compressor tip leakage flow has long been known as the dominant mechanism in determining compressor work input, efficiency and stability [1]. Recent numerical and experimental studies have highlighted the significance of the TLV in the stall inception [2] and the nonsynchronous vibration [3]. Due to the multi-scale nature of the TLV, the understanding of the TLV structure can be further advanced by using scale-resolving simulations and mode decomposition methods.
Mode decomposition methods, such as proper orthogonal decomposition (POD) and dynamic mode decomposition (DMD), are commonly used tools for understanding complex flows by extracting the general spatial and temporal flow structures (i.e., modes) from data [4,5]. Previous studies have demonstrated the capability of mode decomposition methods in identifying the dominant flow physics of compressor flows. Li et al. [6] applied DMD to the DES solution of a high-speed compressor cascade. The interaction between the separation vortex and the corner vortex was captured, which was concluded to be the main cause of the flow field instability. Fu et al. [7] applied DMD to analyze the spike-type stall flow mechanics. The low-frequency perturbation associated with the stall inception was captured, and a reduced-order model for predicting the vortex trajectory and the spike-stall inception location was proposed. Bernhard et al. [8] applied POD and DMD to analyze the surge phenomenon in a centrifugal compressor. It was found that the low-frequency modes account for the filling and emptying processes during surge cycles. Shi et al. [9] used POD to investigate the unsteady behavior of wakes in a compressor cascade. POD analysis extracted the most energetic dynamics successfully, and the phase averaged flow field was reconstructed accordingly.

POD extracts coherent structures in turbulent flows by identifying the optimal set of orthogonal modes ranked by their modal energy. However, the physical meaning of each mode may be difficult to interpret, because the mode shape involves flow field information at different frequencies. DMD extracts the frequency-resolved flow structure with each mode having a characteristic frequency of oscillation and a growth/decay rate. However, DMD lacks a general mode ranking criteria, making it challenging to determine the dominant modes. Recently, a novel modal analysis method named spectral proper orthogonal decomposition (SPOD) [10] has been proposed. It combines the advantages of both POD and DMD: the modes are ranked according to their energy level with each mode having a characteristic frequency. The SPOD method has been successfully applied to extract the spatial-temporal modes of jet and wind turbine flows [11–13]. In this paper, its application in compressor tip leakage flow is explored.

The motivation of this paper is to unveil the modal component of compressor tip leakage flow by using SPOD. This understanding will help turbulence modeling, reduced-order modeling and flow control of compressor tip leakage flow. In the following, the methodology including the flow solver, the SPOD method and the case studied will be detailed first. Afterward, the flow solutions and the SPOD analysis of the near-tip axial velocity and the blade surface pressure will be presented and discussed.

2 METHODOLOGY

2.1 Flow Solver

This study is based on the in-house hybrid RANS/LES flow solver HADES. It uses a node-centered finite volume scheme with an edge-based data structure. The numerical fluxes are solved by the Jameson-Schmidt-Turkel scheme [14] with a Roe’s matrix [15], leading to second-order accuracy in space except for the vicinity of a shock. The second-order implicit time integration scheme is applied for temporal discretization. For simulation of turbulent flows, the standard Spalart-Allmaras turbulence model [16] and its variants [17] are available for the RANS branch, and the standard DDES method [18] and its upgrades [19] are available for the hybrid RANS/LES branch. Validations of HADES can be found in previous works [17, 19].

In this work, a variation of the SA-based DDES method [19] is used. Compared to the original formulation [18], an alternative grid spacing $\Delta = F_{KH} \Delta_{hyb}$ [20, 21] is adopted to unlock the physics of Kelvin-Helmholtz instability. A hybrid low-dissipation scheme [22] is
also used to reduce the fourth-order dissipation term in the LES region.

### 2.2 Spectral Proper Orthogonal Decomposition

In this study, SPOD is applied to identify the spatio-temporal coherent structure within the compressor tip leakage flow. SPOD is a variant of POD [23] that is designed for statistically stationary flows. It can capture the energy-ranked and mutually orthogonal modes at each frequency. Mathematically, the SPOD modes are eigenvectors of the cross-spectral density (CSD) matrix at each frequency, and the eigenvalues correspond to the energy of each mode at a distinct frequency. The procedure of the SPOD method is summarized as follows.

- A matrix containing the spatio-temporal flow data is constructed first. Let the vector \( q_k \in \mathbb{R}^{N_q} \) represent the \( k \)th time snapshot subtracting the time-averaged data, and its length \( N_q = N_g N_v \) where \( N_g \) and \( N_v \) are the number of grid points and investigated variables, respectively. By assembling the snapshot flow field data in chronological order, the spatio-temporal data matrix is obtained:

\[
Q = [q_1, q_2, \cdots, q_N_t] \in \mathbb{R}^{N_q \times N_t} \tag{1}
\]

where \( N_t \) is the total number of snapshots.

- The data matrix \( Q \) is then segmented into \( N_b \) (\( N_b \ll N_t \)) blocks by applying the Welch periodogram method [24], with each block consisting of \( N_f \) snapshots. Each block is assumed to be a statistically independent realization under the ergodicity hypothesis. The discrete Fourier transform (DFT) is performed on each block to convert the problem to frequency domain. To avoid spectral leakage, each block is windowed and overlapped with neighboring blocks. The resultant matrix for the \( k \)th block is:

\[
\hat{Q}^{(k)} = [\hat{q}^{(1)}_k, \hat{q}^{(2)}_k, \cdots, \hat{q}^{(N_f)}_k] \in \mathbb{C}^{N_q \times N_f} \tag{2}
\]

where \( N_f \) is equivalent to the number of resolved frequencies.

- Next, the frequency-domain matrices of blocks are reshaped according to the frequency. For the \( k \)th frequency, the corresponding matrix is:

\[
\hat{Q}_k = [\hat{q}^{(1)}_k, \hat{q}^{(2)}_k, \cdots, \hat{q}^{(N_b)}_k] \in \mathbb{C}^{N_q \times N_b} \tag{3}
\]

The weighted CSD matrix for the \( k \)th frequency is therefore obtained as:

\[
S_k = \frac{1}{N_b} W^{\frac{1}{2}} \hat{Q}_k \hat{Q}_k^* W^{\frac{1}{2}} \in \mathbb{C}^{N_q \times N_q} \tag{4}
\]

where \( W \) is the weight matrix for scaling of different flow variables.

- Finally, the eigenvalue decomposition is performed on the weighted CSD matrix \( S_k \) for each frequency:

\[
S_k = \Phi_k \Lambda_k \Phi_k^* \tag{5}
\]

\[
\Phi_k = [\phi_k^{(1)}, \phi_k^{(2)}, \cdots, \phi_k^{(N_b)}] \in \mathbb{C}^{N_q \times N_b} \tag{6}
\]

\[
\Lambda_k = \text{diag}(\lambda_k^{(1)}, \lambda_k^{(2)}, \cdots, \lambda_k^{(N_b)}) \in \mathbb{R}^{N_b \times N_b} \tag{7}
\]
The eigenvector matrix $\Phi_k$ represents the SPOD mode shapes at the $k_{th}$ frequency. These modes are ranked according to their corresponding eigenvalues $\Lambda_k$, which can be physically interpreted as the disturbance energy of the mode.

The schematic of the SPOD method is shown in Fig. 1. The PYTHON script of SPOD written for this work is open-accessed\(^1\). Detailed theory and derivation of the SPOD method can be found in [10, 13].

![Schematic of the SPOD algorithm.](image)

**2.3 Case Description**

The axial compressor investigated in this paper is the BUAA Stage B rotor from Du et al. [25], whose specifications are summarized in Table 1. The rotor flow has been investigated experimentally using five-hole probe and stereoscopic particle image velocimetry (SPIV) measurements under the stage environment. The measurement locations of SPIV are illustrated in Fig. 2(a), including 10 slices between 10% and 100% of blade tip chord near the suction surface (SS) and 11 slices between 5% and 105% of blade tip chord near the pressure surface (PS).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hub to tip radius ratio</td>
<td>$r_h/r_t$</td>
<td>0.6</td>
</tr>
<tr>
<td>Tip gap to chord ratio</td>
<td>$\tau/c_t$</td>
<td>1.8%</td>
</tr>
<tr>
<td>Tip aspect ratio</td>
<td>$h/c_t$</td>
<td>1.2</td>
</tr>
<tr>
<td>Flow coefficient</td>
<td>$u_1/U_t$</td>
<td>0.58</td>
</tr>
<tr>
<td>Tip Mach number</td>
<td>$M_{1t}$</td>
<td>0.17</td>
</tr>
<tr>
<td>Reynolds number</td>
<td>$Re_{r}$</td>
<td>$1.4 \times 10^4$</td>
</tr>
<tr>
<td>Number of blade</td>
<td>$Z$</td>
<td>17</td>
</tr>
</tbody>
</table>

To simplify the numerical model, only one blade passage of the rotor is considered. The flow domain and the boundary conditions are illustrated in Fig. 2(b). At the inlet, the measured

\(^1\)Package available from: [https://github.com/HexFluid/spod_python](https://github.com/HexFluid/spod_python). [Accessed in Dec 2020]
radial profile of total pressure, total temperature and velocity directions are prescribed. Since
the measured freestream turbulent kinetic energy (TKE) is of two orders of magnitude smaller
than those generated by the Kelvin-Helmholtz instability in the main TLV core, no additional
turbulent perturbations are given at the inlet boundary. A choked nozzle [26] is attached to the
rotor exit, whose throat area is adjusted to match the measured near-stall static pressure over
the upper 20% span at the rotor exit. Euler equations are solved in the nozzle domain to reduce
the calculation cost. A mixing plane is used at the interface between the rotor domain and the
nozzle domain.

The rotor domain is meshed by a hexahedral grid with 3.7 million points, which is generated
by AUTOGRID [27]. The number of mesh points on the spanwise direction, pitchwise direction
and blade surfaces is 117, 97 and 305 respectively. In order to accurately resolve the tip leakage
flow, over 50% of the mesh points are located above the 80% span, including 29 layers inside
the tip gap. The average $y^+$ value for the first-layer mesh points is 1.1, and the linear-law
branch of the standard wall function [28] is used to calculate the shear stress at those points.
A zoom-in view of the rotor mesh is presented in Fig. 2(c). The nozzle domain is meshed by
a hexahedral grid with 0.2 million points, which is generated by an in-house PYTHON script.
The non-dimensional time step is set as $\Delta t \cdot \text{BPF} = 1/300$. This time step corresponds to a local CFL number of $\Delta t \cdot U_t / \Delta t = 1/3$, which is finer than the recommended value of unity [29]. Within each time step, 30 Newton iterations are performed so that the residual of density drops over 2 orders of magnitude. The simulation was performed over 5 rotor revolutions to reach a statistically steady-state, and all snapshots from the last 2 revolutions were used for time-averaging.

3 FLOW FIELD VISUALIZATION

3.1 Transient Flow Field

The transient flow field of the investigated rotor is illustrated in Fig. 3, where the vortical structures are identified by the iso-surfaces of Q-criterion. A large-scale TLV is formed immediately after the flow detaches from the leading edge. Afterward, it breaks down into smaller eddies near the mid-pitch and impinges the PS of the adjacent blade. Part of the main TLV flow passes through the tip gap of the adjacent blade and forms a secondary TLV, which detaches from the mid-chord SS and propagates towards the rotor exit. The impingement and the detachment of the TLVs exert unsteady forces on the blade.

![Figure 3: Snapshot iso-surfaces of $Q = (U_t/\tau)^2/50$ colored by $IQ_v$.](image)

![Figure 4: Power spectral densities of axial velocity fluctuations at 90% span and 50% pitch.](image)

To check the quality of the DDES solution, the resolution quality index $IQ_v$ [30] is plotted on the iso-surfaces of Q-criterion in Fig. 3. The higher the $IQ_v$, the finer the resolution. The results show that the $IQ_v$ value near the main and the secondary TLV region is larger than 0.80, indicating good resolution of DDES. To verify the DDES resolution further, the power spectrum density of the fluctuation component of axial velocity is examined in Fig. 4. Data presented in this plot come from 6 numerical probes evenly distributed between 0% and 100% chord at 90% span and 50% pitch. It shows that DDES can resolve the inertial subrange of the turbulence spectrum down to the frequency of 10 BPF, which again demonstrates the good quality of the DDES result.

3.2 Time-Averaged Flow Field

The time-averaged flow quantities from the experiment and DDES are compared in Fig. 5. For the streamwise vorticity contour in Fig. 5(a), both the experiment and DDES capture qualitatively the same trajectory of the main TLV core featured by high values of vorticity. Due to the breakdown of the main TLV, its vorticity dissipates rapidly near the entry of the
compressor passage. A quantitative comparison on the trajectory of the main TLV core is made in Fig. 6(a), where the location of the TLV core is defined by the maximum vorticity magnitude. It demonstrates that DDES is quantitatively predictive on the main TLV trajectory.

For the TKE contour in Fig. 5(b), both the experiment and DDES capture large TKE values near the trajectory of the main TLV. These high TKE values are produced in the shear layers near the main TLV due to the Kelvin-Helmholtz instability. The TKE values are then dissipated after entering the compressor passage, where the vortex breakdown of the main TLV occurs. This vortex breakdown phenomenon creates small-scale vortices that enhance turbulence dissipation. The distributions of TKE along the main TLV trajectory and the mid-pitch and 95% span line are compared in 6(b). It shows that DDES can qualitatively capture the trend of TKE distribution including the peak of TKE near 50% tip chord.

4 MODE DECOMPOSITION

In this section, the SPOD method is performed to extract and analyze the modal components of compressor tip leakage flow. The first case investigates the normalized axial velocity $u/U_t$ at the mesh points between 80% span and the blade tip. This quantity is an indicator of flow reversal and thus is relevant to the compressor aerodynamic instability (e.g., stall and surge). The second case investigates the normalized pressure $p/\frac{1}{2} \rho_1 U_t^2$ on the blade surfaces. It reflects the unsteady forces exerted by the TLVs on the blade and thus is relevant to the compressor aeroelastic instability (e.g., nonsynchronous vibration).
For both cases, snapshot data are collected every 6 time steps (i.e., sampling frequency $f_s/BPF = 50$), which cuts off the high-frequency components of the sub-grid scales. The normalized control volume of each grid point $V_{i N_g}/\sum_{i=1}^{N_g} V_i$ is used as the weight in Eq. 4, which guarantees the contribution of each grid point to the modal energy is proportional to the mass contained in the control volume. Details about the data and the SPOD parameters for both cases are listed in Table 2.

Table 2: Parameters of databases and spectral estimation of the SPOD analysis.

<table>
<thead>
<tr>
<th>Database</th>
<th>SPOD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case</td>
<td>N_g</td>
</tr>
<tr>
<td>Near-tip axial velocity</td>
<td>$u/U_t$</td>
</tr>
<tr>
<td>Blade surface pressure</td>
<td>$p^{1/2}/p_1 U_t^2$</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>N_f</td>
</tr>
</tbody>
</table>

4.1 Near-Tip Axial Velocity

The SPOD modal energy spectrum of the near-tip axial velocity is shown in Fig. 7(a). In general, the near-tip flow exhibits a broadband characteristic without any spikes in the spectrum, and the majority of the flow energy is contained in the large-scale low-frequency components of the flow. A low-rank behavior at frequencies below 3.0 BPF is observed for Mode 1, which contains a significantly large portion of the energy and thus plays a dominant role. For Mode 2, a hump between 0.4 BPF and 2.0 BPF is observed in the spectrum where the energy contained in Mode 1 is in the same order as that in Mode 1. To illustrate the features of Mode 1 and Mode 2 further, the energy fraction for the $k_{th}$ frequency of Mode $m$, i.e., $\lambda_k^{(m)}/\sum_{j=1}^{N_b} \lambda_j^{(j)}$, is plotted against the frequency in Fig. 7(b). At 0.4 BPF, the energy fraction of Mode 1 is 66%, and over 90% of the total energy at this frequency can be recovered by the leading 9 modes out of the total of 25 modes. At 1.2 BPF, the energy fractions of Mode 1 and 2 are 37% and 16%, respectively. The sum of the leading 12 modes can recover over 90% of the total energy at this frequency. For higher frequencies such as 5.1 BPF, however, the energy fraction of Mode 1 is only 13%, and the leading 20 modes are needed to reconstruct 90% of the total energy at this frequency.

Figure 7: SPOD mode energy for the normalized near-tip axial velocity: (a) energy spectrum and (b) energy fraction at each frequency.
To explain the physical relevance of the above SPOD modes, their mode shapes are plotted in Fig. 8. At 0.4 BPF, the shape of Mode 1 shows two coherent wavepackets: the dominant wavepacket is near the SS leading edge, which corresponds to the swing motion of the main TLV around its time-averaged trajectory; the second wavepacket is near the PS, which corresponds to the unsteady impingement of the main TLV on the PS. The shape of the less energetic Mode 2 is characterized by similar features as Mode 1. However, the wavepacket near the PS has a shorter wavelength than Mode 1, and the wavepacket near the SS extends further downstream compared to Mode 1. At 1.2 BPF, the shape of Mode 1 again identifies the wavepacket of unsteady impingement near the PS, but its wavelength is smaller than that in the 0.4 BPF results in accordance to the higher frequency. The shape of Mode 2 shows a wavepacket near the SS mid-chord due to the unsteadiness of the secondary TLV. At 5.1 BPF, both Mode 1 and Mode 2 show a similar small-scale wavepacket near the SS, which comes from both the secondary TLV and the boundary layer of the SS. The number of small-scale structures found near the PS is far less than those near the SS. The above observations suggest that the main TLV is mainly composed of low-frequency fluctuations, leading to the low-rank feature of Mode 1. On the other hand, the secondary TLV has components across all frequencies due to its interaction with the SS boundary layer. Its components between 0.4 BPF and 2.0 BPF are pronounced, leading to the hump in the spectrum of Mode 2.

Figure 8: SPOD mode shape (real part) of the normalized axial velocity.

4.2 Blade Surface Pressure

The SPOD modal energy spectrum of the blade surface pressure is illustrated in Fig. 9(a). Due to the strong coupling between velocity and pressure in incompressible flows, a low-rank phenomenon for Mode 1 and an energy spectrum hump for Mode 2 are also observed here. The energy fraction of each mode is shown in Fig. 9(b). At 0.4, 0.8 and 1.2 BPF, the energy fraction of Mode 1 is 58%, 45% and 49%, respectively. At 0.8 BPF, Mode 2 also contains 18% of the total energy at this frequency. These numbers are qualitatively similar to those in the velocity analysis.

The SPOD mode shape of the blade surface pressure is presented in Fig. 10. For the PS, a wavepacket is observed in the Mode 1 shape at all frequencies, which represents the impingement of the main TLV on the PS. Its wavelength decreases with increasing the frequency. Mode 2 also shows a wavepacket pattern, but it has a multilobe structure in
the streamwise direction. Both modes are localized above 80% span except for the high frequencies components. For the SS, a wavepacket is also observed in both modes and all frequencies, which corresponds to the detachment of the secondary TLV and the boundary layer. The affected spans are higher than 95%, which is narrower than its counterpart on the PS. For the Mode 2 shape at 0.8 BPF, a weak structure can be observed downstream the mid-chord above the 80% span. This structure is correlated with the secondary TLV only, and it leads to the hump in the Mode 2 spectrum. The above observation suggests that the main TLV contributes more to the unsteady blade force than the secondary TLV, which is as expected since the secondary TLV is triggered by the main TLV.
5 CONCLUSIONS

In this paper, the unsteady modal component of compressor tip leakage flow has been analyzed for the first time by using the SPOD method. The investigated case is a low-speed compressor rotor, and its flow field data are obtained by DDES simulations that are validated against the SPIV measurements. Several conclusions can be drawn as follows.

The near-stall tip leakage flow in the investigated low-speed compressor rotor is governed by the main TLV. It forms immediately after detaching from the leading edge and then impinges on the PS of the adjacent blade. Part of the main TLV flow passes through the tip gap in the mid-chord of the adjacent blade, leading to a secondary TLV. It is these TLVs that affect the aerodynamic/aeroelastic stability of the rotor.

The SPOD modal analysis on the axial velocity field reveals that both TLVs have a broadband characteristic. The main TLV is mostly composed of low-frequency (i.e., $f/BPF < 3.0$ in this case) components, and its coherent structures are contained in Mode 1. The secondary TLV has both low-frequency and high-frequency components due to its interaction with the SS boundary layer. Its coherent structures at low frequencies are contained in Mode 2.

The SPOD modal analysis on the blade surface pressure shows that the unsteady force due to the impingement of the main TLV is more significant than that created by the detachment of the secondary TLV, in terms of the affected span and the magnitude of the mode shape. It suggests that reconstruction of the aerodynamic blade forces is possible by using only the leading mode of the low-frequency component of the blade pressure.

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References


