INFLUENCE OF THE REACTION ON THE PERFORMANCE OF THE CROSSFLOW TURBINE

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ABSTRACT

The crossflow turbine is a hydraulic turbomachine that is frequently considered to be an action turbine, i. e., one where there is no pressure variation between the rotor inlet and outlet. However, it is well known that this turbine may develop some degree of reaction, for some operating conditions, namely, for large values of the blade-jet velocity ratio. In this paper the consequences of this fact are explored using a simple 1D model for the water flow. The results of the modelling explain the typical reduction of water flow rate this turbine exhibits for large blade-jet velocity ratios, and show that the peak efficiency values can be affected, and decreased by the onset of the reaction, depending on the geometric parameters being used. This study shows the utility of this refined analysis that take into account the reaction, for the correct choice of the geometric parameters of a crossflow turbine, eventually leading to improved designs, with better performance.

KEYWORDS

TURBOMACHINERY, CROSSFLOW TURBINE, BÁNKI-MICHELL TURBINE, 1D MODEL, SMALL HYDROPOWER PLANTS

NOMENCLATURE

b	width
D	diameter
D_2/D_1	diameter ratio
g	acceleration of gravity
H	hydraulic head
k_n	nozzle loss coefficient
k_r	rotor loss coefficient
p	static pressure
p_o	stagnation pressure
Q	volume flow rate
U	blade or transport velocity
U_1	rotor tip speed
U_1/V_0	blade-jet velocity ratio
V	absolute velocity
V_0	jet velocity
W	relative velocity
z	elevation or height
Greek letters	
α	absolute flow angle
β	relative flow angle
ΔI	energy losses at the rotor

η	hydraulic efficiency
ρ	density
φ	nozzle entry angle – see Fig. 1a)
χ	fraction of losses occurring during the first passage of the rotor
Subscript	
0	Station at inlet to the nozzle of the turbine
1	Station at inlet to the first passage through the rotor
2	Station at exit to the first passage through the rotor
3	Station at inlet to the second passage through the rotor
4	Station at exit to the second passage through the rotor
in	relative to station 0
ref	relative to station 4, inside the turbine enclosure

INTRODUCTION

The crossflow turbine is a hydraulic partial admission turbine where the water crosses the region of the rotor blades twice. In the first passage, after leaving the nozzle, the water goes from the rotor outside to an empty space defined by the blades, while the second passage takes the water from the inside space to the outside, see Fig. 1a). The main advantage of this turbine is the ease of manufacture, which leads to a low cost of production. However, the value of its peak efficiency tends to be lower than what is achievable with the other hydraulic turbines, restricting its application to instances with low power. To minimize this drawback, both experimental and theoretical studies have been conducted in the past.

As examples of the experimental strategy the works of Nakase et al. (1982), Khosrowpanah et al. (1988), and Olgun (1998) can be mentioned. In this approach, the turbine efficiency is experimentally measured, as a function of the rotor rotational speed, using different geometries (diameter ratio, number of blades, rotor blade angle at the outer and inner diameter, nozzle exit angle, etc.). Besides demonstrating that the turbine geometric parameters have a strong influence on the peak efficiency values, these experimental studies showed conclusively that the crossflow turbine tends to behave like an action turbine for the smaller values of the rotational speed, but as this parameter is increased, some degree of reaction sets in, i.e., the water flow through the rotor starts to be accompanied by a decrease in pressure. The different geometric parameters influence the point where the reaction begins to appear.

On the theoretical front, there are examples of the use of computational codes to calculate the two-dimensional flow inside the turbine like De Andrade et al. (2011), Sammartano et al. (2013) or Leguizamón and Avellan (2020). Nevertheless, this approach tends to be quite expensive in terms of computational resources, and so does not allow the screening of large number of different geometric configurations.

This discussion indicates the desirability of an alternative approach, requiring modest computational efforts. One such approach consists on the use of one-dimensional models to analyze quickly and inexpensively the dependency of the peak efficiency value with the turbine geometric parameters. Historically, this was the first strategy utilized to choose the geometric parameters of a crossflow turbine, see, for instance, Mockmore and Merryfield (1949), Varga (1959) and Haimerl (1960). These references also provide some experimental data that back up and validate the models proposed. A more recent work (Pereira and Borges 2017) also proposed a one-dimensional model for the flow of water through the turbine, with detailed supporting experimental evidence.

All the above-mentioned one-dimensional models assume the turbine to behave like an action turbine, i.e., consider the water flow inside the rotor to be at constant pressure. However, as discussed above, this assumption fails to be true for some operating conditions. To address this shortcoming, a one-dimensional model of the flow through a crossflow turbine that considers the reaction is described in the present paper. This model is a refined version of the model described in

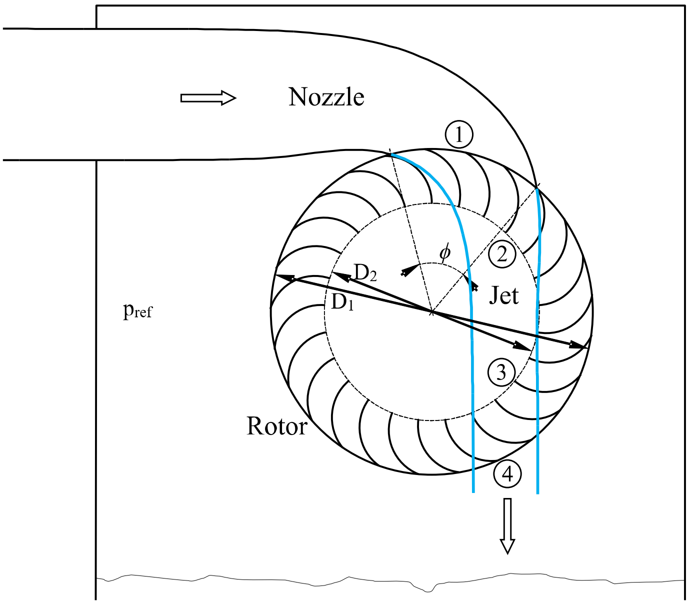

Figure 1(a): Sketch of a typical Crossflow turbine

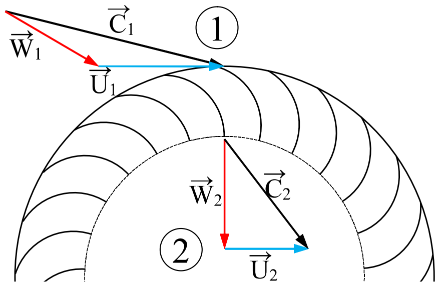

Figure 1(b): Velocity triangles relative to the blade geometry, for the first passage

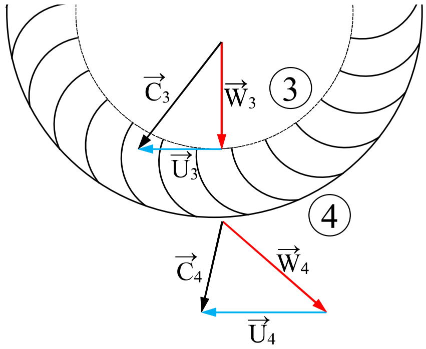

Figure 1(c): Velocity triangles relative to the blade geometry, for the second passage

Pereira and Borges (2017). The results of this modelling show that the peak efficiency values can be affected by the reaction, depending on the geometric parameters being used. This study shows the utility of this refined analysis for the correct choice of the geometric parameters of a crossflow turbine, eventually leading to improved designs, with better performance. In the next section of this paper the one-dimensional model will be presented, and the relevant equations will be derived. The results obtained will be shown and discussed on the following section, and afterwards the final conclusions will be drawn.

THEORY – 1D MODELLING

The flow through the crossflow turbine will be assumed steady, incompressible and one-dimensional, and several stations will be considered along the flow path through the turbine. In fact, station 0 will be considered to be the section at inlet to the nozzle, while station 1 will be at the exit of the nozzle, which is simultaneously the inlet to the rotor. Station 2 is at the end of the first passage through the rotor blades, followed by station 3, at inlet to the second passage and station 4 at exit of the same passage, which also constitutes the exit from the rotor (see Fig. 1a), Fig. 1b) and Fig. 1c)). The velocities triangles commonly assumed at stations 1, 2, 3 and 4 are presented in Fig. 2, and also Fig. 1b) and Fig. 1c). The velocity triangles at station 2 and station 3 are equal. In fact, the absolute velocity at station 3 is equal to the absolute velocity at station 2 because the flow only suffers a translation, and the blade transport velocity remains the same at these stations ($\vec{U}_2 = \vec{U}_3$) because the radius at station 2 is equal to the radius at station 3. For the same reason, it is also $\vec{U}_1 = \vec{U}_4$. The absolute flow angle, α_1, and the relative flow angle, β_1, are indicated in the top sketch of Fig. 2, concerning the overall velocity triangles. Between station 1 and 4 the effect of body forces will be neglected, which is equivalent to assume that the elevation or height above a given reference datum at 4 is approximately equal to the corresponding value at station 1 ($z_4 \cong z_1$).

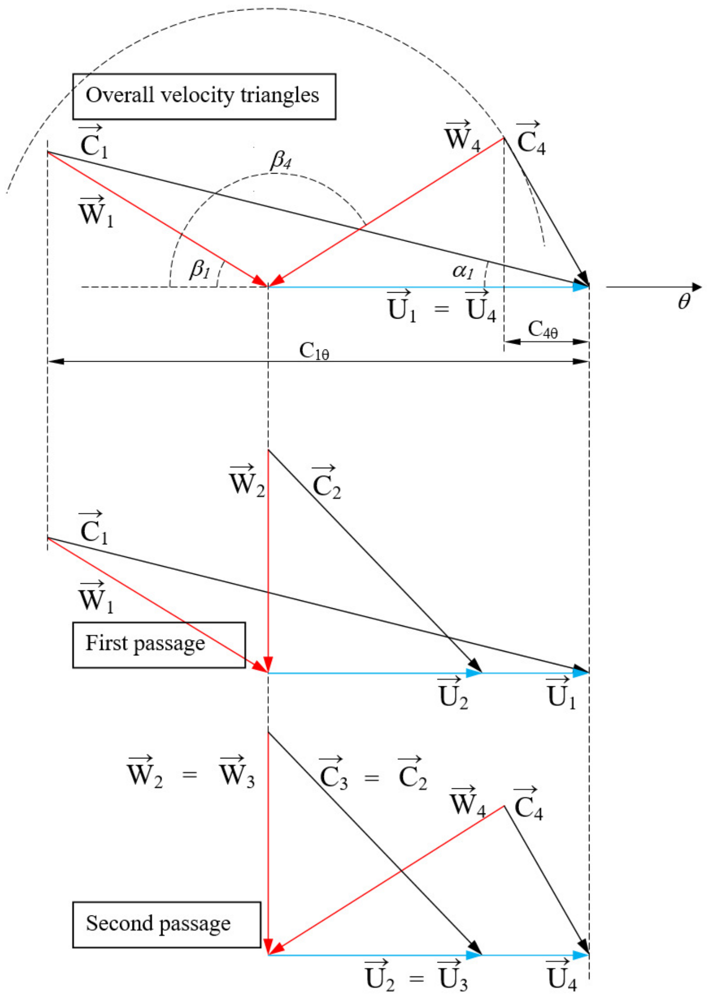

Figure 2: Velocity triangles at rotor stations 1, 2, 3 and 4

The hydraulic head applied to the turbine as a whole is given by:

$$\rho g H = \left[\left(p_0 \right)_{in} - p_{ref} \right] \tag{1}$$

where $\left(p_0 \right)_{in}$ is the stagnation pressure at inlet to the turbine and p_{ref} is the static pressure inside the turbine enclosure, where the kinetic energy can be neglected, and at the same elevation of the rotor. By definition, the jet velocity will be:

$$V_0 = \sqrt{2gH} \tag{2}$$

Both the head and jet velocity do not depend on the rotor rotational speed and on the losses and, for that reason, these values will be used to normalize some of the variables appearing latter on.

Applying the one-dimensional energy equation between the turbine inlet and station 1 (exit of the nozzle, inlet to the rotor), where the static pressure is p_1, the velocity is C_1, and considering that Δp_0 is the drop in stagnation pressure occurring inside the nozzle, one gets:

$$(p_0)_{in} = p_1 + \frac{1}{2}\rho C_1^2 + \Delta p_0 \tag{3}$$

which implies that:

$$\frac{(p_0)_{in} - p_{ref}}{\rho} = \frac{p_1 - p_{ref}}{\rho} + \frac{1}{2}C_1^2 + \frac{\Delta p_0}{\rho} \tag{4}$$

This relation will be written as:

$$\frac{p_1 - p_{ref}}{\rho} = \frac{1}{2}\left[k_n^2 V_0^2 - C_1^2\right] \tag{5}$$

where k_n is a loss coefficient that takes into account the losses that occur at the turbine nozzle, giving a reduction of stagnation pressure inside the nozzle amounting to:

$$\frac{\Delta p_0}{\rho} = \left[1 - k_n^2\right]\frac{V_0^2}{2} \tag{6}$$

In the case $k_n = 1$, there will be no losses at the turbine nozzle.

In a similar way, the rotor losses will be described by the equation:

$$\Delta I = \left[1 - k_r^2\right]\frac{W_1^2}{2} \tag{7}$$

where, again, $k_r = 1$ for the no losses case at the rotor. Both k_n and k_r lie in the interval $[0,1]$.

It is generally accepted that the reaction sets in when the rotor is unable to swallow at station 2, the water flow entering at station 1 of the rotor, see Varga (1959), Van Dixhorn et al. (1984), Borges and Pereira (2004) and Pereira (2007). This causes a pile up of the water leading to an alteration of the pressure at inlet to the rotor. In fact, the value of the pressure p_1 increases, and as a consequence the water flow rate decreases, adjusting to the amount that can be swallowed at station 2. Since this increase in pressure, or onset of reaction, is linked to what happens between station 1 and 2, it is essential to study the velocity triangles involved in the first passage through the rotor. This will be achieved applying again the one-dimensional energy equation to the relative flow through the first passage across the blade region, taking into account that the static pressure at station 2 will be equal to p_{ref}, and assuming that the losses during this passage are equal to a fraction χ of the overall rotor losses. The final expression is:

$$\frac{p_1 - p_{ref}}{\rho} = -\frac{W_1^2 - U_1^2}{2} + \frac{W_2^2 - U_2^2}{2} + \chi\Delta I \tag{8}$$

Equating the right-hand sides of equations (4) and (8), it follows that:

$$\frac{1}{2}\left[k_n^2 V_0^2 - C_1^2\right] = -\frac{W_1^2 - U_1^2}{2} + \frac{W_2^2 - U_2^2}{2} + \chi \Delta I \qquad (9)$$

Making use of the trigonometric relation $W_1^2 - U_1^2 = C_1^2 - 2C_1 U_1 \cos\alpha_1$ (see Fig. 2 where the velocities and angle α_1 are shown), equation (9) can be cast into the form:

$$(\sin\alpha_1)^2 \left(\frac{D_1}{D_2}\right)^2 .C_1^2 + 2U_1 \cos\alpha_1 .C_1 - U_2^2 - k_n^2 V_0^2 + 2\chi\Delta I = 0 \qquad (10)$$

which can be interpreted as a second order equation on the unknown C_1. Solving for the unknown, one obtains:

$$C_1 = \frac{-U_1 \cos\alpha_1 + \sqrt{U_1^2 \cos^2\alpha_1 - \left(\frac{D_1}{D_2}\right)^2 (\sin\alpha_1)^2 .\left[2\chi\Delta I - k_n^2 V_0^2 - U_2^2\right]}}{\left(\frac{D_1}{D_2}\right)^2 (\sin\alpha_1)^2} \qquad (11)$$

where only the positive root was considered.

If the C_1 value given by equation (11) is greater than $k_n V_0$, it would imply that $p_1 < p_{ref}$ (see eq. (5)), which is physically unrealistic. Therefore, when the previous condition applies, it means that equation (11) should not be used, and the turbine behaves like an action turbine, with $p_1 = p_{ref}$. In this case, the efficiency should be calculated using the expression presented in reference Pereira and Borges (2017).

Now applying the mass balance of water between station 1 and 2, assuming the passages between blades run full and that the blade blockage is negligible, there results:

$$Q_1 = Q_2 \quad \Rightarrow \quad C_1 \sin(\alpha_1).b_1.\varphi\frac{D_1}{2} = W_2.b_2.\varphi\frac{D_2}{2} \qquad (12)$$

where b_1 and b_2 are the nozzle and rotor widths respectively (usually it is $b_1 \cong b_2$ − the width is the dimension normal to the plane of Fig. 1a)), φ is the nozzle entry angle, and D_1 and D_2 are the diameters at inlet and exit to the first passage through the rotor blade region, respectively – see Fig. 1a).

Substituting equation (11) into (12) and assuming $b_1 = b_2$, leads to:

$$W_2 = -\frac{U_2}{\tan(\alpha_1)} + \sqrt{\frac{U_2^2}{(\sin\alpha_1)^2} + k_n^2 V_0^2 - 2\chi\Delta I} \qquad (13)$$

Normalizing eq. (13) and (12), these two equations are transformed into:

$$\frac{W_2}{V_0} = -\frac{1}{\tan(\alpha_1)}\frac{U_2}{V_0} + \sqrt{\frac{1}{(\sin\alpha_1)^2}\left(\frac{U_2}{V_0}\right)^2 + k_n^2 - \frac{2\chi\Delta I}{V_0^2}} \qquad (14a)$$

$$\frac{C_1}{V_0}=\frac{1}{\tan(\alpha_1)}\frac{D_2}{D_1}\frac{W_2}{V_0} \tag{14b}$$

The amount of energy exchanged in the rotor is given by Euler´s Turbomachinery equation as:

$$\eta gH = U_1(C_{1\theta}-C_{4\theta}) \tag{15}$$

and from the velocity triangles presented in Fig. 2, it is possible to conclude that:

$$\eta gH = U_1[C_1\cos\alpha_1 - U_1 + W_4\cos\beta_1] \tag{16}$$

Using the one-dimensional energy equation between station 1 and 4, and simplifying it, it is possible to relate the magnitude of the velocity W_4 with the velocity magnitudes at station 1 by:

$$\frac{W_4^2-U_1^2}{2}+\Delta I=\frac{1}{2}\left[k_n^2V_0^2-C_1^2\right] \tag{17}$$

and replacing here the expression $W_1^2-U_1^2=C_1^2-2C_1U_1\cos\alpha_1$, already used, it follows that:

$$W_4=\sqrt{k_n^2V_0^2+U_1^2-2U_1C_1\cos\alpha_1-2\Delta I} \tag{18}$$

Introducing eq. (18) into eq. (16), implies that:

$$\eta gH = U_1\left[C_1\cos\alpha_1 - U_1+\cos\beta_1\sqrt{k_n^2V_0^2+U_1^2-2U_1C_1\cos\alpha_1-2\Delta I}\right] \tag{19}$$

This relation can be made non-dimensional, taking into account that $2gH=V_0^2$, obtaining as a consequence:

$$\eta=2\frac{U_1}{V_0}\left[\frac{C_1}{V_0}\cos\alpha_1-\frac{U_1}{V_0}+\cos\beta_1\sqrt{k_n^2+\left(\frac{U_1}{V_0}\right)^2-2\frac{U_1}{V_0}\frac{C_1}{V_0}\cos\alpha_1-\frac{2\Delta I}{V_0^2}}\right] \tag{20}$$

This expression for the efficiency should be used when the turbine behaves like a reaction turbine, i.e., for $p_1>p_{ref}$. If $p_1=p_{ref}$, the expression presented in reference Pereira and Borges (2017) should be used, as indicated previously. For the sake of completeness, this last expression is repeated here:

$$\eta=2\frac{U_1}{V_0}\left[k_n\cos\alpha_1-\frac{U_1}{V_0}+k_r\cos\beta_1\sqrt{k_n^2+\left(\frac{U_1}{V_0}\right)^2-2k_n\frac{U_1}{V_0}\cos\alpha_1}\right] \tag{21}$$

Equation (20) (together with eq. (11)) allows the calculation of the turbine efficiency as a function of the turbine geometric parameters (α_1, β_1 and $\frac{D_2}{D_1}$), loss coefficients (k_n and k_r), and the blade-jet velocity ratio, $\frac{U_1}{V_0}$, which characterizes the rotational rotor speed.

RESULTS

The volume flow rate, Q_1, through the rotor can be worked out as:

$$Q_1 = C_1 \sin(\alpha_1).b_1.\varphi\frac{D_1}{2} \tag{22}$$

where b_1, φ, and D_1 were already defined. If one non-dimensionalizes this relation using the volume flow rate when the turbine behaves like an action turbine, which is a constant value given by $(Q_1)_{action} = k_n V_0 \sin(\alpha_1).b_1.\varphi\frac{D_1}{2}$, the following expression is attained:

$$\frac{Q_1}{(Q_1)_{action}} = \frac{1}{k_n}\frac{C_1}{V_0} \tag{23}$$

This relation is depicted in Fig. 3 as a function of the blade-jet velocity ratio, for the case of no losses ($k_n = k_r = 1$), and a turbine with commonly used geometric parameters, namely, $\alpha_1 = 17°$, $\beta_1 = 30°$, and $\frac{D_2}{D_1} = 0.667$.

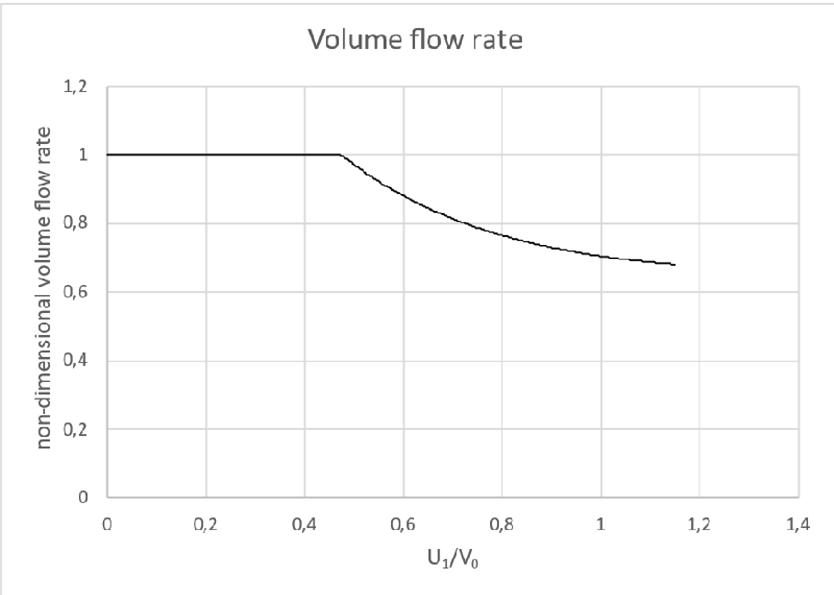

Figure 3: Evolution of the volume flow rate as a function of blade-jet velocity ratio

The constant portion of the curve equal to 1.0 corresponds to the interval of operating conditions for which the turbine behaves like an action turbine. It is seen that the turbine behaves like an action turbine for the smaller values of the blade-jet velocity ratio, while for larger values of this parameter there is some reaction, and the volume flow rate decreases as a consequence. The larger the blade-jet velocity ratio, the more pronounced is the effect of the reaction, and the decrease in the volume flow rate. The onset of reaction occurs for a value of blade-jet velocity ratio around 0.5, which is the value for which the model predicts the peak efficiency will occur. Therefore, it is to be expected that the value of peak efficiency may be affected by the reaction.

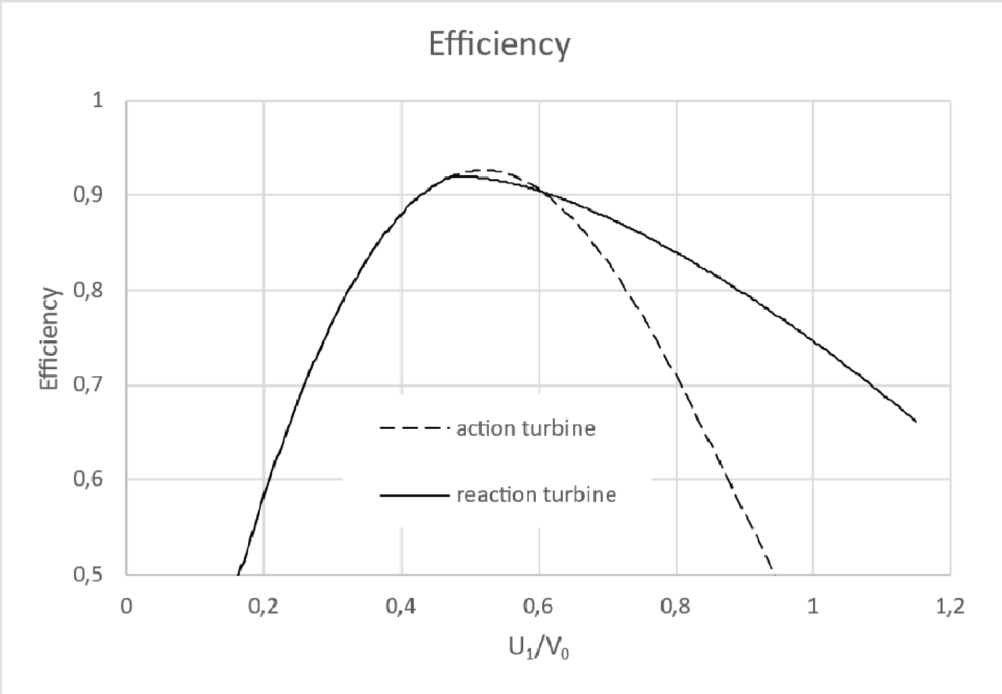

Figure 4: Comparison of the efficiency as a function of blade-jet velocity ratio, for action and reaction turbine (no losses, $k_n = k_r = 1$)

This conclusion is confirmed by the next plot, Fig. 4, that shows the evolution of the efficiency as a function of the blade-jet velocity ratio for both the cases of action (dashed line) and reaction turbine (solid line), using the same parameters considered in Fig. 3 (namely, $\alpha_1 = 17°$, $\beta_1 = 30°$, and $\frac{D_2}{D_1} = 0.667$, and no losses). The new model predicts an efficiency which is higher that the value advanced by the action turbine model, except for the interval of blade-jet velocity ratio between the onset of reaction ($\frac{U_1}{V_0} = 0.47$) and a blade-jet velocity ratio of $\frac{U_1}{V_0} = 0.61$, for the example being discussed. Inside this interval, the reaction turbine model predicts a lower value of efficiency. Since peak efficiency for the action turbine model occurs also inside this interval (see the dashed line in Fig. 4), it can be stated that the peak efficiency predicted by the reaction turbine model is smaller than the value obtained modelling the turbine as an action turbine.

Comparing the model predictions with the experimental results presented in Van Dixhorn et al. (1984) and Pereira (2007), it is concluded that the general trend of the predictions agrees well with the one shown by the experimental results, in the sense that it is seen experimentally a reduction of the water volume flow rate that gets more pronounced for larger values of the blade-jet velocity ratio.

The value of the blade-jet velocity ratio at the onset of reaction as a function of some of the geometric parameters (namely, α_1, β_1, and $\frac{D_2}{D_1}$) is reported in Fig. 5. This plot indicates that the smaller is the ratio of $\frac{D_2}{D_1}$, the smaller will be the value of the blade-jet velocity ratio at the onset of reaction, for a given value of the exit nozzle angle, α_1. It also shows, again, that the onset of reaction is usually close to the values leading to peak efficiency ($\frac{U_1}{V_0} \cong 0.5$), or even smaller. This implies the need of analyzing the reaction, if one wants to improve the crossflow turbine performance, because for the geometric parameters usually found in the design of these turbines, the peak efficiency is probably affected by the reaction. The above theory also predicts that the inlet rotor angle, β_1, does not influence the values of the blade-jet velocity ratio at the onset of the reaction.

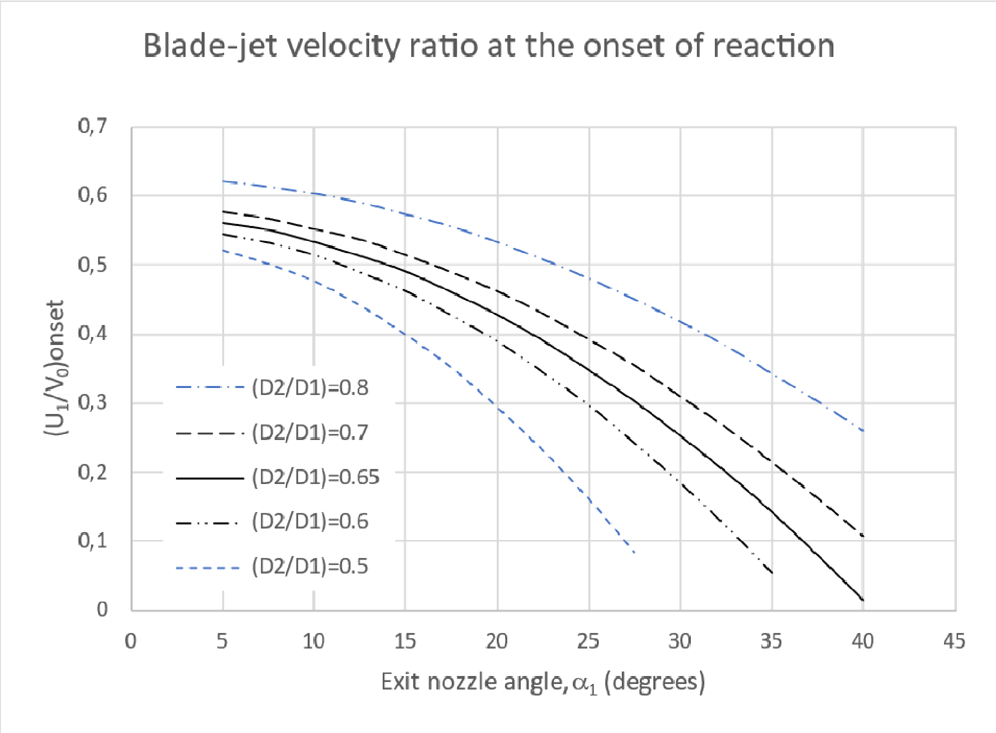

Figure 5: Blade-jet velocity ratio at the onset of reaction as a function of the turbine geometric parameters

The results of Pereira and Borges (2017) suggest that the geometric parameters used for plotting Fig. 3 and Fig. 4 are not the ideal ones. The search for a set of geometric parameters that could lead to improved efficiencies should be driven by the reaction turbine model just presented since it predicts the peak efficiency values more closely to reality.

CONCLUSIONS

A one-dimensional model for the crossflow turbine that takes into account the reaction that occurs for some of the rotor rotational speeds was described in this paper.

The starting point for the analysis is the observation that the reaction sets in when the rotor is unable to swallow all the water flow entering the rotor, at exit of the first passage though the bladed region. This condition changes the magnitude of the velocity, and the pressure value at inlet to the rotor, and influences the turbine efficiency. Mathematical relations were derived for the magnitude of the velocity at inlet to the rotor and for the change caused to the turbine hydraulic efficiency when the turbine behaves like a reaction turbine.

The theory results indicate that the reaction is more pronounced for larger values of the blade-jet velocity ratio, in agreement with the experimental trend. It was shown that for the geometric parameters commonly used in the design of this type of turbines, the value of the peak hydraulic efficiency is reduced due to the reaction. A discussion of the influence of some geometric parameters on the value of the blade-jet velocity ratio at the onset of reaction was presented.

Finally, this work proves the desirability of accounting for the reaction in order to choose correctly the geometric parameters of a crossflow turbine, eventually leading to improved designs, with better performance.

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