DESIGN OF SUPERSONIC AXIAL TURBINE FOR WASTE HEAT RECOVERY FROM HEAVY INDUSTRIAL FLUE GASES THROUGH AN ORC SYSTEM

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ABSTRACT
The present study focuses on the design of a high-efficient Organic Rankine Cycle (ORC) power production system utilizing waste heat from heavy industrial exhaust gases. During the design process of these systems priorities are given to the turbine, where the optimization aims at the maximization of the power output. This target can be achieved easily by considering supersonic turbines.

Towards to this direction, a single-stage turbine was designed, where the stator is considered as a convergent – divergent nozzle. A new design methodology is introduced for the convergent sector, while the Method of Characteristics is used for the divergent sector. In addition the impulse rotor design was selected, applying the corner method flow.

The proposed stator has an enthalpy loss coefficient of 14.5%, while the rotor has respectively 11.3%, leading to a turbine of 111.85 kW power output and 82.65% total to total isentropic efficiency.

KEYWORDS
ORC, SUPERSONIC TURBINE, METHOD OF CHARACTERISTICS, CORNER FLOW METHOD, ENTROPY RISE BUDGET

NOMENCLATURE

Abbreviations
EOS Equations of State
MLN Minimum Length Nozzle
MOC Method of Characteristics
ORC Organic Rankine Cycle
PRSV Peng – Robinson – Stryjek – Vera
SST Shear Stress Transport

Symbols
\( b_{\text{loss}} \) Bending energy loss
\( f_{\text{loss}} \) Friction energy loss
\( h \) Specific enthalpy
\( k \) Polytropic Index
\( M \) Mach number
\( P \) Pressure
\( R_1 \) Stator inlet circular radius

Greek Letters
\( \theta \) Local Flow Angle
\( \mu \) Mach angle
\( \nu \) Prandlt – Meyer angle

Subscripts
\( \text{conv} \) Convergent
\( \text{cr} \) Critical
\( \text{ps} \) Pressure side
\( \text{ref} \) Reference
\( \text{ss} \) Suction side

INTRODUCTION
Organic Rankine Cycles (ORC) are considered to be a very favorable technology, for the exploitation of low-temperature heat sources in the field of power production from waste heat
recovery, due to the use of low-boiling point organic working fluids (Zhao et al., 2012). The cycle consists of an evaporator, a turbine (expander), a condenser and a pump. The design point of these applications is the maximization of net – work output (Colonna et al., 2015, Efstathiadis et al., 2015). The most important component is the turbine. Considering the manufacturing and maintenance costs, more compact, with fewer stages, turbines are preferred in ORC applications (Gori et al., 2012), whereas organic working media with high density are widespread used. Thus, supersonic turbines with larger pressure ratio should be modelled for power density and efficiency enhancement.

The optimization of turbine’s geometry is the main factor, who affects the efficiency of the turbine, which highly dependents the overall efficiency of the ORC cycle (Reinker et al., 2015). In order to achieve supersonic regime, stator blade is designed as a convergent - divergent nozzle. The convergent sector is designed following an optimization process which targets to entropy rise minimization. The design procedure of the divergent sector and rotor is based on the Method of Characteristics (MOC), a numerical procedure, based on the Euler equations, usually employed for solving 2-D compressible flow problems (Anderson, 2014). Entropy rise is mainly due to friction and bending losses (as the flow follows the blade stagger angle direction), in the convergent sector and contingent shock losses in the divergent part.

Concerning the rotor design, it has been proposed that impulse rotor blades (with close to zero degree of reaction) are the optimal rotor geometry, for the maximization of the turbine’s power – output. The rotor blades are formed as circular arcs, which are produced by a MOC based method, called “corner flow method”. The stator and rotor blade geometries are combined into a developed design tool that can employ the optimized thermodynamic results from the commercial software and automatically produce geometries for different working fluids.

HEAT SOURCE AND CYCLE ANALYSIS

Heat Source
In this work the heat source contains flue gases produced from a magnesite mining plant, called Grecian Magnesite (Chalkidiki, Greece). The production process leads to the generation of heavy industrial exhaust gases of 150 °C and flow of 21.389 m³/s, which consist of: N₂: 65.7%, CO₂: 16.8%, H₂O: 10.3 % and O₂: 7.2 %.

Cycle Analysis
The Organic Rankine Cycle examined in this analysis is illustrated in figure 1a. and it is about a simple ORC cycle, without recuperator, whose optimization target is the maximization of the network output. During the cycle analysis, we considered pump isentropic efficiency at 90%, turbine isentropic efficiency at 85% and mechanical efficiency 98% respectively (Macchi and Astolfi, 2016). Also heat losses from the turbine, piping and pump were neglected and the system was modeled at steady state.

Figure 1. a) Organic Rankine Cycle layout, b) Turbine net-power and thermal efficiency for different compositions of isopentane – isobutane mixture
**Working fluid selection**

Several different working fluids were examined for the investigation of the behavior of ORC system. The thermodynamic calculations were validated by commercial software (ASPEN Plus) and a reference fluid property library. Among the different working fluids, we concentrated in the mixture of Isobutane – Isopentane, as it was found that it has the greatest potential for net-power maximization. Six combinations of this mixture (form 90% isopentane and 10% isobutane to 40% and 60% with 10% step size) were tested, and the results are shown in figure 1b. It can be deduced that the net-power reaches a maximum value of 115.03 kW in the composition of 30% isobutane and 70% isopentane. This is also the point with the lowest thermal efficiency, which has to do with the increase of the heat from the source, in order to achieve the maximum net-work. Since the objective function of the simulation is the net-power output, the 30% isobutane – 70% isopentane mixture is selected for the turbine design.

**GOVERNING EQUATIONS**

Dense working fluid gases, in ORC applications, usually operate in thermodynamic conditions, near the critical point (thermodynamic non – simple region). These conditions are characterized by complex thermodynamic behavior that deviates significantly from the ideal one. In this work we calculated the appropriate thermodynamic quantities through a reference fluid property library (NIST-REFPROP library ver. 9.1.), which was incorporated in a programming platform (MATLAB R2019a). The reference library calculates thermodynamic states near the critical point of the gases, via multi – parameter equations of state, such as the Peng – Robinson – Stryjek – Vera (PRSV) equation, and presents accuracy of the order of 10^{-3} regarding the experimental data (Lemmon et al., 2002). One major challenge of these EOS is the calculation of thermodynamic properties, such as internal energy, enthalpy and entropy. In case of enthalpy, it can be defined according to equation 1 (Colonna and Silva, 2003):

\[
h - h_{ref} = \int_{T_{ref}}^{T} C_p^{ig}(T) \, dT + \int_{P_{ref}}^{P} \left[ v - T \frac{\partial v}{\partial T} \right] \, dP \quad (1)
\]

where \( C_p^{ig}(T) \) is the specific heat capacity in constant pressure, calculated by the ideal gas law. The first part, of the right – hand side of the equation, is the enthalpy calculated by the ideal gas model and the second part is called the enthalpy energy departure, which is the difference between the real and ideal case. Internal energy and entropy can be calculated following the same procedure.

The working fluid, which was selected for the analysis, is the mixture of 30% isobutane – 70% isopentane, as from the cycle analysis it proved that leads to the highest power output production. Its main thermodynamic properties are listed in table 1.

<table>
<thead>
<tr>
<th>Working Fluid</th>
<th>( P_c ) [MPa]</th>
<th>( T_c ) [K]</th>
<th>( M_w ) [kg/kmol]</th>
<th>( \omega )</th>
</tr>
</thead>
<tbody>
<tr>
<td>isobutane</td>
<td>3.629</td>
<td>407.81</td>
<td>58.12</td>
<td>0.184</td>
</tr>
<tr>
<td>isopentane</td>
<td>3.378</td>
<td>460.35</td>
<td>72.15</td>
<td>0.227</td>
</tr>
<tr>
<td>30% isob – 70 isop</td>
<td>3.654</td>
<td>445.66</td>
<td>67.94</td>
<td>0.214</td>
</tr>
</tbody>
</table>

Table 1. Thermodynamic properties of the selected mixture and its pure fluid components

**Mixing Rules**

The selection of the isobutane – isopentane mixture forces us to the slight transformation of the dense gas EOS, using the Wong-Sandler (WS) mixing rules, in order to recalculate the constants a(T) and b, from the PRSV EOS (Stryjek and Vera, 1986). According to the aforementioned rule (eq. 2), these constants are calculated:

\[
b = \sum x_i b_i \quad (i), \quad a = \sum x_i x_j a_{ij} \quad (ii), \quad a_{ij} = (a_i a_j)^{0.5} (1 - k_{ij}) \quad (iii)
\]
In equation 3iii, $k_{ij}$ is called binary interaction parameter, and it is identical for each pure compound of the mixture, and slightly depends on the temperature.

**STATOR BLADE DESIGN**

Turbines, used in ORC applications, must be designed in such way making them attractive, in terms of cost, by heavy industries, in order to be these systems applicable. A cost-effective solution is to achieve the appropriate pressure ratio, in only one – stage turbine. This can be carried out by supersonic turbines. In such a turbine, the flow acceleration from subsonic to supersonic regime, takes place in stator, while the power output is produced in the rotor. Thus, the proposed stator design is influenced by the convergent – divergent nozzle and it is illustrated in figure 2. The flow enters the blade, as incompressible and highly subsonic in the flow inlet ($ab'$), and then accelerates in the convergent sector to reach Mach unity in the blade throat ($c'g$), where the supersonic acceleration takes place (Eleutheriou et al., 2016). The flow exits the blade ($d'e$) at high supersonic speed.

![Stator Design](image)

**Figure 2. Proposed supersonic stator design**

**Subsonic Convergent Sector**

Firstly, the flow enters at horizontal direction the flow inlet sector, which is designed as a circular arc of radius $R_1$. The flow accelerates from $M_{in}$ to $M_{ab'}$, and depending these Mach numbers the radius $R_1$ is calculated through the continuity equation. Then the flow continues, in the convergent sector, where it accelerates from $M_{ab'}$ to $M^* = 1$ and changes direction from horizontal to the stagger angle. In order to achieve this flow turning, the lower surface of the convergent sector is designed as a circular arc of radius $R_2$, and the upper surface is calculated through the passage diameter, which is dependent from the Mach number, as the flow accelerates towards the throat. The major challenge in the design of convergent sector is the choice of the radiuses $R_1$ and $R_2$, which are calculated through an optimization process. The objective function of the optimization is the minimization of the entropy increase, as it is shown in the equation 3.

$$\min(\Delta s) = f(R_1, R_2, M(x))$$

The entropy increase in the expansion process is due to the flow losses. The basic flow losses in this sector are the friction losses and the bending losses. Both of these are actually flow kinetic losses, for their preliminary calculation the ensuing process is followed: At first we divide the convergent passage into $N$ elementary small control volumes. The flow in every control volume
accelerates from velocity $V_i$ to $V_{i+1}$ and it is turning at $\delta \theta$ angle, and so the passage diameter is reduced from $d_i$ to $d_{i+1}$. The kinetic losses, per unit mass, due to friction and bending are calculated through equation 4, where the friction factor $f_i$ is calculated through the correlation in equation 5 (Fang et al., 2011). The bending loss factor $k_b$ by fittings based on experimental data. In our case, considering the small turning angle in every step, loss factor $k_b$ varies from 0.0139 to 0.0015, for values of $R_2/d_i$ of 0.5 to 10 respectively.

$$ f_{\text{loss},i} = f_i \frac{LV_i^2}{D} \frac{2}{2} = f_i \frac{2\pi R_2}{d_i} \frac{\delta \theta}{\epsilon} \frac{V_i^2}{2} \quad (i), \quad b_{\text{loss},i} = k_{b,i} \frac{V_i^2}{2} \quad (ii) \quad (4) $$

$$ f_i = 1.613 \left[ \ln \left( 0.234 \frac{\epsilon}{d_i}^{1.1007} - \frac{60.525}{R_{\text{exit}}^{1.1105}} + \frac{56.291}{R_{\text{exit}}^{1.0712}} \right) \right]^{-2} \quad (i), \quad k_{b,i} = f \left( \frac{R_2}{d_i}, \delta \theta \right) \quad (ii) \quad (5) $$

In a closed adiabatic system, the kinetic losses are transformed into heat, which is absorbed from the flow increasing its enthalpy. So the actual real enthalpy can be assessed from a variation of the energy equation, showed in equation 6.

$$ h_{\text{real},i} = h_{\text{i},i} + f_{\text{loss},i} + b_{\text{loss},i} \quad (6) $$

For a given pressure, increasing enthalpy leads to the increase of the irreversibilities of the process that means entropy increase. These two basic loss factors are competitive each other: if we want to reduce the bending losses we can increase the radius $R_2$, which will increase the total blade length and thus increasing the friction losses and vice versa. The last one indicates that there is an optimal radius ratio $R_2/R_1$, where these two losses are minimized. This ratio is defined by implementing the golden section search method, as the main algorithm for the optimization process.

Another major aspect during the optimization process is the Mach number distribution in the passage. The proposed Mach distribution is depicted in figure 3a. At the first the value of $M_{ab}$ is chosen, from which the inlet radius $R_1$ is calculated. This Mach number is desired to be as low as possible, because it reduces the friction losses, however its value is constrained by the optimization constrains discussed below. The Mach number distribution in convergent sector is considered as a power function, which is depicted in equation 7. Again the minimization of friction losses is the key factor of choosing this type of function, as we want to delay the rapid acceleration as much as possible close to the throat, in order to reduce these losses. The parameters of the function $(m_1,m_2,n)$ are defined through a 3x3 equation system, where $M_{\text{conv}}(x_{i=1}) = M_{ab}$, $M_{\text{conv}}(x_{i=N}) = 1$ and also the derivative $M'_{\text{conv}}(x_{i=1}) = dM/dx|_{ab'}$.

$$ M_{\text{conv}}(x) = m_1 (x - x_0)^n + m_2 \quad (7) $$

**Optimization Constrains**

The major constrain during the optimization process is the blade thickness limitation, which is affected by manufacturing restrictions. Considering the trailing edge as the point of minimum blade thickness, a limitation of minimum diameter of 0.5 mm is imposed. The blade thickness at any point must not exceed this limitation. A better way of understanding this constrain is the depiction of circle envelope of the blade in figure 3b. According to this constrain any circle, illustrated in the figure, must have a diameter that is higher 0.5 mm.

The diameter distribution of the circle envelope affects the blade passage diameter, thus it affects the Mach number distribution and the value of $M_{ab}$. It turns out that for a given radius ratio $R_2/R_1$, the minimum value of $M_{ab}$ which fulfills the described constrain, provides the optimal Mach number distribution, concerning the entropy rise. Incorporating this into the optimization process, we finally get the optimal values for $R_1, R_2, M(x)$. 


Supersonic Divergent Sector

The divergent sector of the stator is consisted from the supersonic nozzle. The last one can be divided in two types; namely:

i. Gradual expansion nozzles (Fig. 4, a), which are used mostly in experimental applications for a high-quality flow at desired exit conditions (e.g. supersonic wind tunnels).

ii. Minimum-length nozzles (MLN) (Fig. 4, b), which are used in commercial applications due to weight and length restrictions (e.g. supersonic turbines, rocker nozzles).

For the proposed stator blade, the minimum-length nozzle (MLN) method was chosen for the divergent sector, due to the fact that gradual expansion nozzles are characterized by high manufacturing accuracy, so any design error could further reflect the expansion waves than originally expected, leading to higher exit velocity than the nominal one. Additionally, MLNs are preferable in turbomachines because they are adaptive to weight and size limitations.

Both of the design nozzle methods are based on the method of characteristics (MOC). The latter is a numerical procedure employed 2-D, inviscid, steady, supersonic flow problems, relying on the governing flow equations being mathematically "hyperbolic". For a 2-D flow, irrotational flow is assumed as well (Khan et al., 2013). The basic principle of this method are the characteristic lines, whose slopes are calculated through equation 8.

\[
\frac{dy}{dx}_{\text{char}} = \tan(\theta \pm \mu)
\]  

\( (8) \)
In equation 9, \( \vartheta \) is the local flow angle and \( \mu = \arcsin(M^{-1}) \) is the Mach angle. The governing equation is depicted in equation 9i, which can be rewritten using the Prandlt–Meyer function, as it is depicted in equation 10ii.

\[
d\vartheta = \pm \sqrt{M^2 - 1} \frac{dV}{V} \quad (i) \quad \vartheta \pm \nu(M) = \text{const.} \quad (ii)
\]  

In the previous equations, sign + or – indicates the left-running and right-running characteristic lines. In case of MLNs, the maximum angle \( \vartheta^* \), for a known \( M_{ex} \), is \( \vartheta^* = \nu(M_{ex})/2 \). The Prandlt–Meyer function, varying polytropic index \( k \) and the Mach number, is calculated through equation 10. In case of \( k < 1 \), the imaginary number \( z = -i \) is added, so the angle \( \nu(M) \) corresponds to real number (Wheeler and Ong, 2013).

\[
\nu(M) = \begin{cases} 
\sqrt{\frac{k+1}{k-1}} \tan^{-1}\left(\sqrt{\frac{k+1}{k-1}} (M^2 - 1)\right) - \tan^{-1}(\sqrt{M^2 - 1}) & , \quad k > 1 \\
\cot^{-1}(\sqrt{M^2 - 1}) + \sqrt{M^2 - 1} + \frac{\pi}{2} & , \quad k = 1 \\
\frac{1}{z} \tanh^{-1}(z \cdot \sqrt{M^2 - 1}) - \tan^{-1}(\sqrt{M^2 - 1}) & , \quad k < 1
\end{cases}
\]  

After the implementation of the MOC the resulting nozzle geometry is connected with the trailing edge with the line \( de \). The proper blade design must consider that the slope of the line \( de \) \( (\vartheta_{de}) \) must be the same with the slope of the nozzle that means the stagger angle \( (\theta_s) \). If \( \vartheta_{de} > \theta_s \), then expansion waves are formed, at the nozzle exit, leading to undesirable further expansion and thus higher exit Mach number than the designed one. If \( \vartheta_{de} < \theta_s \), then a shock wave is formed at the nozzle exit, which increases significantly the irreversibilities in the expansion process and thus reducing stator’s isentropic efficiency. These two unfavorable cases are shown in figure 5.

**Figure 5. Typical examples of unfavorable cases, after the nozzle exit**

The slope of the line \( de \) \( (\vartheta_{de}) \) is dependent of the stator’s pressure ratio, and consequently the ratio of Mach numbers in stator’s inlet and outlet. For a fixed exit Mach number, the latest statement indicates that there is a single inlet Mach number, for which the slope of the line \( de \) is equal with the stagger angle. This inlet Mach number also affects radius \( R_1 \), thus it is taken into consideration in the optimization process.

**ROTOR BLADE DESIGN**

The stator blade row, which was described in the previous section, is coupled with an impulse rotor blade row. The design of the blade is the impulse rotor, as it has been proven that rotor blades with close to zero degree of reaction present the higher work coefficient (Dick, 2015). Towards to this direction, an MOC based method, called “corner flow method”, is used to design such impulse rotor blades in the current work (Kladovasilakis et al., 2017). (Figure 6).
Corner Flow Method

According to this method, the front portion of the suction side is curved in such way that generated shock waves are canceled on the concave surface of the pressure side and then a parallel flow initiates (Paniagua et al., 2014). Therefore, the flow experiences a corner flow expansion, where the expansion waves are canceled also by curved surface, until parallel flow is achieved at the exit. As, the entirety of governing equations in steady supersonic conditions are hyperbolic, the MOC is implemented as a corresponding numerical method. Finally, using this method, the flow turning is limited and there is zero loading in the center section of the passage.

A vortex flow field is applied throughout the circular arcs of the rotor. The governing equation for such fluid domain is given by the equation 11:

$$V \cdot R = \text{constant}$$  \hspace{1cm} (11)

The flow velocity is \( V \) and \( R \) is the radius in the vortex field. Divide equation 8 by the critical speed \( V_{cr} \) and by radius of sonic velocity streamline \( r^* \), and taking into account that \( R^* = R/r^* \) and \( M_{cr} = V/V_{cr} \), it can be compressed to:

$$M_{cr} \cdot R^* = 1$$  \hspace{1cm} (12)

For the calculated values of relative inlet and outlet Mach numbers and flow angles (\( \beta \)) of the rotor, and have the Mach numbers \( M_{ps} \) in pressure and \( M_{ss} \) in suction side selected, as a design option, we can obtain the critical Mach numbers and the respective sonic radius, for each case, from the equation 16. Therefore, the turning angles in the transition arcs (\( \Theta \)) and the respective ones in the circular arc region (\( \alpha \)) can be found from equations 17 and 18:

$$\theta_{i,j} = \pm [v(M_{cr.i}) - v(M_{cr.j})] \quad i = ps, ss \quad j = in, out$$  \hspace{1cm} (13)

$$\alpha_{i,j} = \beta_{j} - \theta_{i,j} \quad i = ps, ss \quad j = in, out$$  \hspace{1cm} (14)

In equation 14, the (-) sign corresponds to the pressure side (ps) and the (+) sign corresponds to the suction side (ss) of the blade.

TURBINE DESIGN RESULTS AND DISCUSSION

Combining the pre-described design approaches, an algorithm, which produces the geometries of the supersonic turbine blades (rotor and stator), was developed for specific operating conditions.

The algorithm was tested for the selected mixture of 70% iso-pentane and 30% isobutane and the operating conditions and obtained results for the turbine stage is shown in table 2. In addition, the optimized stator geometry, containing the key parameters is described in table 3.
Table 2. Proposed turbine stage operating conditions

<table>
<thead>
<tr>
<th>Property</th>
<th>Stator Inlet</th>
<th>Stator outlet</th>
<th>Rotor inlet</th>
<th>Rotor outlet</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature [K]</td>
<td>394.5</td>
<td>340.47</td>
<td>340.47</td>
<td>336.33</td>
</tr>
<tr>
<td>Pressure [MPa]</td>
<td>1.000</td>
<td>0.124</td>
<td>0.124</td>
<td>0.104</td>
</tr>
<tr>
<td>Mach number (absolute /relative)</td>
<td>0.0651</td>
<td>2.000</td>
<td>1.250</td>
<td>1.380</td>
</tr>
<tr>
<td>Flow angle [deg] (absolute /relative)</td>
<td>0.00</td>
<td>72.54</td>
<td>61.31</td>
<td>62.36</td>
</tr>
</tbody>
</table>

Table 3. Results of the optimization process for the stator geometry.

The aforementioned design algorithm is validated with commercial simulation software (ANSYS). The simulation was performed in a specialized solver, for turbomachinery applications (CFX). The turbulence model applied is k-omega SST (Shear Stress Transport) with the energy equation, as it has been proved it has the best accuracy in supersonic regime regarding the computational cost (Bufi et al., 2015, Medeiros et al., 2014). Also, the near wall treatment provided the simulation with a value of $y^+$ less than 1. The validation of the optimization process is performed via examining 3 cases of the convergent sector of the stator (low, high and optimal ratio $R_2/R$). The two dimensional mesh has an average of 150,000 mainly elements for each case of the stator blade. In figure 7 (a-c) Mach number contour plots are depicted for each case, while in figure 7d and 7e, the corresponding maximum local and average entropy rise is shown.

![Mach number contour plots](image)

Figure 7. Mach number contour plots for a) Low ratio $R_2/R_1$, b) Preliminary optimal case and c) High ratio $R_2/R_1$; d) Local entropy rise, for each case, in pressure and suction side respectively, e) average entropy rise in the meanline.
The numerical analysis reveals that, there is a distinct region near the pressure side of the blade, where incompressible, low Mach number regime prevails. The decelerated flow increases the irreversibilities; leading finally to energy losses, which represent the “bending losses”, and entropy rise. For small values of ratio \( R_2/R_1 \) these “bending loses” prevail increasing the irreversibilities. On the other hand, for larger values of ratio \( R_2/R_1 \), entropy rise is mainly due to friction. The latest leads to the conclusion that there is an optimal value for \( R_2/R_1 \) where the entropy rise is minimize, which in our case has been calculated at 1.664. This optimization is clear in figures 7d and 7e, which represent the local entropy rise in the pressure side and suction side of the convergent part and the average entropy rise in the blade’s meanline for each case.

Beyond the convergent part analysis, the whole stator and rotor blade is investigated under numerical analyses. The two dimensional mesh has more than 200,000 mainly elements in the case of rotor. In figure 8a and 8b Mach number contour plots are illustrated for the stator and rotor respectively.

Expansion waves generated by MOC are clear in case of stator; however additional waves are generated from the trailing edge as well, increasing the velocity to approximate Mach 2.4 locally, as it is show in figure 9a. The latest was expected, as trailing edge is responsible for expansion and shock waves formation, and in combination with manufacturing constrain responsible for the highest entropy rise. The generated shock and expansion waves are illustrated in the pressure gradient contour plot of figure 10a. These waves are responsible for fluctuations of flow velocity in the mean line discharge, as it shown in figure 9a. In table 4, the contribution of each stator blade surface to the entropy increase is shown. The results confirmed that the divergent part contributes the highest percentage of entropy increase (59.95%), especially the trailing edge, which contributes the 33.42% of the total entropy increase. This is confirmed also from the entropy rise contour plot in figure 11a.

Figure 8. a) Mach number contour plot in stator blade, b) Mach number contour in rotor blade, c) Velocity triangles of the turbine stage

Figure 9. a) Mach number distribution in the stator blade, b) Mach number distribution in the rotor blade, c) Pressure distribution in the pressure and suction side of the rotor blade.
The pressure side of the convergent sector also contributes with a significant amount (24.16%), leading finally to an average entropy increase of 39.55 J/kg-K and an enthalpy loss coefficient of 14.5% (Denton, 1993).

Concerning the rotor, oblique shock waves and expansion waves are also observed throughout the rotor passage. As it is observed in figures 8b, 9b and 10b, the flow experiences two shock waves in the inlet, one caused by the upper blade’s leading edge before the inlet (in -0.2 of dimensionless blade span) and another shock wave caused by the lower blade’s leading edge in 0.2 of the span. These shocks are responsible for local a pressure rise in the suction side, as it is shown in 9c. These characteristics of the flow match with the design method. In the blade’s outlet multiple shock and expansion waves are observed causing fluctuations to the flow velocity. The entropy rise due to energy losses is 13.29 J/kg-K, which leads to an enthalpy loss coefficient of 11.4% and a total to total isentropic efficiency of 91.2 %. Comparing with theoretical impulse rotor design, the final geometry deviates slightly, with a degree of reaction of 5.9%, which is clear from the deviation of the expansion line 2-3, from the horizontal theoretical one, in the h-s diagram of figure 9c.

The shock waves that originate at stator trailing edge propagate downstream and eventually interact with the rotor blades. This interaction strongly affects the flow topology in the entire stage and has a strong impact on the stage performance, resulting in unsteady behavior in the rotor blades. However, this may be beyond the scope of this paper. The steady state analysis can be carried out as a preliminary design. Future developments will focus further on this interaction by time – accurate simulations, and evaluate the influence of various design parameters in the shock interaction. (Rubechini et al., 2013).

![Pressure Gradient Contour Plot](image)

**Figure 10. a) Pressure gradient contour plot in stator blade, b) Pressure gradient contour plot in rotor blade**

<table>
<thead>
<tr>
<th>Blade surface</th>
<th>Local Entropy rise [J/kg-K]</th>
<th>Local Entropy rise (percentage)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inlet</td>
<td>2.90</td>
<td>0.71%</td>
</tr>
<tr>
<td>Convergent, ps</td>
<td>161.40</td>
<td>24.16%</td>
</tr>
<tr>
<td>Convergent, ss</td>
<td>62.30</td>
<td>15.19%</td>
</tr>
<tr>
<td>Divergent</td>
<td>108.80</td>
<td>26.52%</td>
</tr>
<tr>
<td>Trailing Edge</td>
<td>137.10</td>
<td>33.42%</td>
</tr>
<tr>
<td>Maximum Local Rise</td>
<td>410.20</td>
<td>-</td>
</tr>
<tr>
<td>Passage Area Average</td>
<td>39.55</td>
<td>-</td>
</tr>
</tbody>
</table>

**Table 4. Stator blade entropy rise budget**
Finally, taking into consideration the stator and rotor energy losses among with the h-s diagram of figure 9c, the turbine’s total to total isentropic efficiency (stator and rotor) is 82.65 %. The resulting efficiency seems very promising, however it leaves room for improvement regarding the shock loses in the divergent part of the stator, the stator’s trailing edge and the rotor’s leading edge. The actual turbine net – power output, is then slight reduced from the respective one, calculated from the cycle analysis, in the value of 111.85 kW.

CONCLUSIONS

In the present work a design approach for waste heat recovery from heavy industrial flue gases through an ORC system is proposed. Cycle optimization results for several different working fluids indicate that using a mixture of 30% isobutane and 70% iso-pentane achieves the maximum net power output of 111.85 kW. The proposed process covers the whole turbine design, from stator’s inlet to the rotor’s outlet, including a new design methodology for the convergent sector of the stator blade. The resulting single-stage turbine reaches a total to total isentropic efficiency of 82.65 %. The main conclusions can be summarized as follows:

- The highest percentage of entropy rise (average and local) in the total stage, takes place in the stator blade (74.85% of the total stage average rise). Concerning the stator’s entropy rise budget, entropy increase in the convergent sector around 39.35% of the maximum local entropy increase (410.20 J/kg-K). This indicates that the convergent sector must be design methodically, and the proposed optimization process aims to this direction.

- The entropy rise in the convergent part results from bending and friction losses. The local entropy rise in the pressure side is higher than the respective one in the suction side. From small ratios of $R_2/R_1$, bending losses are increased, while from higher ratios friction losses prevail. This indicates that there is an optimal value of $R_2/R_1$, where the entropy rise in the convergent part is minimized, which in our case has been calculated to the value of 1.664.

- The greatest contributor to the total stator entropy rise is the divergent sector, which contributes around 59.95%, in total. The trailing edge alone contributes to 33.42%. The manufacturing constrain assists the entropy increase, as it leads to a wide trailing edge, regarding the total blade size.

- Considering the rotor blade, the entropy increase of 13.29 J/kg-K is mainly related to the bow shock waves that appear at the rotor’s inlet and the oblique shock waves that appear at the rotor’s outlet. Due to their presence, a reduction of the relative Mach number is observed in the outlet, which leads to a slight change of the blade’s operating point.
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