FORCED RESPONSE FREEFORM OPTIMIZATION OF A RADIAL TURBINE USING THE ADJOINT METHOD

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ABSTRACT
Radial turbines equipped with inlet guide vanes are subject to a forced excitation: The guide vanes distort the pressure field upstream of the turbine wheel and induce a wake, that excites the blades as they pass. This is problematic as soon as resonance occurs. Hence, one is interested in studying whether and how one may reduce the induced forced response of the blades to avoid high cycle fatigue.

Aiming at a reduction of the resulting forced response, a gradient-based freeform optimization is conducted for the studied radial turbine: Sensitivities are evaluated by means of the adjoint method. Herein, the complex step method is exploited to gather accurate derivatives of both dynamic stiffness matrix and excitation force. The resulting sensitivities are smoothed by means of the Vertex Morphing method before the mesh is updated.

Few optimization iterations suffice to successfully reduce the blades’ most prominent forced response. Hence, the method delivers a promising approach to make radial turbines more robust against guide vane induced excitations. A comparison of both baseline and optimized design illustrates how those have been altered by the optimizer and explains the improvements.

KEYWORDS
TURBOCHARGER, RADIAL TURBINE, FREEFORM OPTIMIZATION, VERTEX MORPHING, ADJOINT SENSITIVITY, FORCED RESPONSE

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INTRODUCTION
Inlet guide vanes upstream of a radial turbine induce pressure disturbances that each of the blades passes as the wheel rotates. This causes an excitation that can resonate with one of the blade’s eigenfrequencies. The arising vibrational deflections may damage and destroy the wheel. Some authors, for example Drozdowski [2011] and Naik et al. [2018], have studied how geometrical changes to the radial turbine blades affect the vibratory response by using a direct approach: They varied the blade by altering single parameters, such as the blade’s thickness, and ran dedicated simulations for each geometry change.

Herein, a different approach is chosen: We use shape sensitivities in order to steer a geometry update to influence the forced response in a desired way. The applied method incorporates that altering a wheel’s shape will alter its eigenfrequencies and thus affects both load and damping at resonance conditions. To keep the simulation cost low, the method avoids CFD simulations. However, this comes of course at the expense of the accuracy of the physical modeling. Publications revolving around gradient-based optimization of radial turbines, for example Müller [2019] or Schwalbach [2021], typically choose a CAD-based parametrization of the geometry as this simplifies all further steps in the design process of a radial turbine wheel. We did not choose this approach for the purposes of this paper: Instead, the nodes on the design surface of the studied geometry serve as parametrization. The consequence of this choice is that we have much more degrees of freedom and do not restrict the optimization too much to a given model parametrization.

In the field of topology optimization, the gradient-based optimization of a forced response is very common, see for example Jog [2002] or Long et al. [2019]. A number of functionals have been proposed to measure a forced response (Niu et al. [2018]). The one we are using here is named dynamic compliance and comes with the particular advantage of being computationally cheap as we will discuss later.

The radial turbine we are studying here has exactly twice as many guide vanes $n_{gv}$ as it has blades $n_b$. Hence, we expect a nodal diameter zero (ND0) mode shape, i.e. all blades vibrate in phase. The excitation frequency $f$ depends on the rotation rate $n_{rps}$ according to $f = n_{gv} \cdot n_{rps}$. Resonance occurs when $f = \lambda_i/(2\pi)$, where $\lambda_i$ is the $i$th angular frequency of the studied turbine wheel.

There are a number of possibilities to influence the fluid-structure-interaction of the blade and the wakes downstream of the guide vanes: Intentional mistuning is a promising path to alter the wheel’s vibration patterns and aerodynamic damping to reduce its response, see Nakos et al. [2021]. Netzhammer et al. [2019] proposed a jet injection at the shroud contour of a radial turbine to reduce the blades’ response and showed its benefits both numerically and experimentally. Another possibility is given by the choice of the number of guide vanes $n_{gv}$: Not only does this choice affect the aerodynamic load at resonance conditions but also the applicable aerodynamic damping, see Beirow et al. [2016]. The approach the authors study in this paper is the application of gradient-based shape optimization techniques to influence how a tuned wheel responds to a certain exciting harmonic load.

SIMULATION MODEL
The finite element model
The finite element method (FEM) is used to discretize the vibration problem. For this purpose, one segment of the radial turbine is meshed using quadratic tetrahedral elements, i.e. each tetrahedron is composed of 4 corner nodes and 6 edge nodes. The edge nodes are implicitly
defined by the position of the respective corner nodes, as we place them right in middle of two corner nodes. Hence, the edge nodes do not move independently throughout the optimization and thus do not constitute additional design variables in our optimization setup. The mesh contains around 110k nodes and 66k elements. It is visualized in fig. 1. The blue faces show what will be used as design surface for the optimization, i.e. the pressure side and the suction side. The white faces are non-design surfaces. The shaft is not modeled. Instead, the turbine is fixed at the red surface. The green faces represent the periodic interface. Note that using a periodic interface instead of cyclic symmetry boundary conditions is a suitable choice for this study, as we are interested in ND0 modes.

The FEM code at hand is a custom implementation in the open-source program SCILAB (Scilab 6.1.0). SCILAB uses an interpreted language and is thus not well-suited to run expensive loops. Yet, it excels at vectorized operations, such as matrix-vector multiplications. Thus, costly loops like the assembly of the stiffness matrix $K$ need to be fully vectorized to attain reasonable performance. Hence, the methods and libraries as presented in Rahman and Valdman [2013] and Valdman [2020] are adapted to the custom code at hand.

The first problem we solve with this code is the general eigenvalue problem

$$\lambda_i^2 M + K \phi_i,$$

that determines the angular frequencies $\lambda_i$ and eigenvectors $\phi_i$ of the geometry at hand. Note that $M$ is the mass and $K$ is the stiffness matrix. The second problem we wish to solve is the harmonic forced response vibration problem with viscous damping given as

$$M \ddot{x}_i + ZD \dot{x}_i + K x_i = F_i \sin(\lambda_i t)$$

for a range of investigated angular resonance frequencies $\lambda_i$. Note that the damping $ZD$ and the excitation $F_i$ will be explained in more detail in subsequent sections. One can transform eq. 2 into the following equivalent linear system of equations

$$(-\lambda_i^2 M + j\lambda_i ZD + K) u_i = F_i \quad \text{or} \quad \tilde{K}_i u_i = F_i,$$

where $j$ is the imaginary unit and $\tilde{K}_i$ is the dynamic stiffness matrix. One can show that $x_i = \text{Im}(u_i e^{\lambda_i t})$ solves eq. 2. Hence, for every possible resonance within the operational range of the radial turbine, we need to solve one additional large, yet sparse linear system of
equations (LSE). As this is costly, we instead approximate \( u_i \) using a common technique in structural dynamics called mode acceleration method (Cornwell et al. [1983]). Let

\[
\Phi = (\phi_1 \quad \phi_2 \quad \ldots \quad \phi_{m-1} \quad \phi_m \quad \varphi) \quad \text{with} \quad m \geq i_{\text{max}}
\]

be a matrix composed of the \( m \) available eigenvectors \( \phi_i \) and \( \varphi = K^{-1}F_{\text{const}} \). Note that \( i_{\text{max}} = 11 \) is the number of resonances we have within the operational range of the turbine. Furthermore, \( \varphi \) constitutes the static response to the steady load \( F_{\text{const}} \). The exact composition of \( F_{\text{const}} \) will be discussed in the next subsection. Using \( \Phi \), we approximate

\[
u_i \approx k_i^\top \Phi \quad \text{with} \quad k_i = (\Phi^\top \tilde{K}_i \Phi)^{-1} \cdot (\Phi^\top F_i).
\]

Using this approximation, we no longer need to solve one LSE per resonance. Additionally, the studies shown in Liu et al. [2015] suggest that using the mode acceleration method is also a good choice in terms of sensitivity calculation as we will conduct here.

**Excitation**

A harmonic force vector of the form

\[
F_i \sin(\lambda_i t)
\]

is used on the right of eq. 2 to excite the radial turbine. Thus, the excitation frequency matches the \( i \)th resonance of the blade. The vector \( F_i \) is modeled as

\[
F_i = F_{\text{const}} \lambda_{i}^3.
\]

Herein, the factor \( \lambda_{i}^3 \) is used scale the applied load vector. The argument is as follows: In practical engine applications, the amplitude of the attacking pressure fluctuations varies with rotation rate. The connection being that high rotation rates are typically connected to higher pressure ratios and mass flows, respectively, leading to high excitation amplitudes. In contrast, small rotational speeds are connected to little aerodynamic load. Thus, the optimization algorithm takes into account that an increase of an eigenfrequency is accompanied by a higher resonance rotation rate and higher excitation, respectively.
The vector $F_{\text{cons}}$ defines the shape of the attacking force field. We intend to avoid costly transient CFD calculations for every resonance and every optimization iteration. Thus, the force vector is simplified to what is visualized in fig. 2: We excite the blade by using a vector force field that attacks the blade at two distinct positions at leading edge.

In order to justify this simplification, we study fig. 3: The shown pressure distribution is the result of an unsteady CFD simulation conducted with Siemens STAR-CCM+: The frequency-domain based Harmonic Balance solver has been applied to gather transient results and extract both the phase and amplitude of the guide vane order induced pressure on the blade's surface. The rotation rate and pressure ratio have been chosen to reflect maximum engine load. Fig. 3 highlights that the peak pressure amplitude is typically found close to the leading edge of the blade. This observation motivates the simplification of the force field to two vectors attacking the blade at leading edge as shown in fig. 2.

**Damping**

The determination of the damping of a vibrating radial turbine is subject to current research. While the structural damping seems to play a negligible role in integral turbine wheels, the damping induced by the surrounding gas appears to be dominant, see Weber and Kühhorn [2019]. To capture these effects, Beirow et al. [2016] proposed to model the aerodynamic damping by exploiting one dimensional acoustic wave propagation theory: Presume a harmonically vibrating face $A$ acting as acoustic source, that emits into the far field. Then the fluctuating pressure $p_{\text{fluc}}$ on the surface is proportional to the velocity $\dot{x}$ of the face, i.e.

$$p_{\text{fluc}} = \rho a \dot{x} = Z \dot{x},$$

where $\rho$ is the gas density, $a$ its speed of sound and $Z = \rho a$ the so-called acoustic impedance. Hence, the viscous damping force acting on that face $A$ is

$$F_{\text{visc}} = Z A \dot{x}.$$  

Thus incorporating this into our model eq. 2 requires the assembly of a matrix $D$: This diagonal matrix contains non-zero entries only for nodes sitting on the geometry’s surface and stores appropriate area information of the respective surrounding element faces. Scaling this matrix with a reasonably chosen acoustic impedance of $Z = 340 \text{kg/}(\text{m}^2 \text{s})$ thus yields an aero-acoustically motivated damping model for our studies. Having determined angular frequencies $\lambda_i$ and eigenvectors $\phi_i$, we may also use $Z D$ to estimate the modal damping $\zeta_i$ using

$$\zeta_i = \frac{Z \phi_i^\top D \phi_i}{2 \lambda_i}.$$  

Note that we do not require $\zeta_i$ for the calculations within the optimization. However, we will use it to study how the shape changes throughout the optimization affect the applied damping.
SENSITIVITY CALCULATION

The adjoint method

The primal problem for the $i$th resonance in residual form is given by

$$\tilde{K}_i u_i - F_i = 0,$$  \hspace{2cm} (11)

where $\tilde{K}_i$ is the complex and symmetric dynamic stiffness matrix, $F_i$ is the load vector and $u_i$ refers to the state variable. Furthermore, the cost function we wish to differentiate and minimize is the so-called dynamic compliance

$$\bar{f}_i := |f_i| = |F_i^\top u_i|$$ \hspace{2cm} (12)

Using this particular cost function $\bar{f}_i$ to measure the forced response will prove beneficial in terms of computational cost, as it saves the solution of the adjoint linear system of equations. We start by noting that

$$\frac{d\bar{f}_i}{dX} = \frac{d|f_i|}{dX} = \frac{1}{|f_i|} \left( \text{Im}(f_i) \cdot \text{Im} \left( \frac{df_i}{dX} \right) + \text{Re}(f_i) \cdot \text{Re} \left( \frac{df_i}{dX} \right) \right).$$ \hspace{2cm} (13)

With that information, we thus may focus on calculating the gradient $\frac{df_i}{dX}$, where $X$ represents the considered design space, i.e the nodal positions of the given finite element discretization. Applying the chain rule, we get

$$\frac{df_i}{dX} = \frac{\partial f_i}{\partial X} + \frac{\partial f_i}{\partial u_i} \frac{du_i}{dX}. $$ \hspace{2cm} (14)

Differentiating eq. 11 w.r.t. $X$ delivers

$$\frac{d\tilde{K}_i}{dX} u_i + \tilde{K}_i \frac{du_i}{dX} - \frac{dF_i}{dX} = 0 $$ \hspace{2cm} (15)

Rearranging eq. 15 for $\frac{du_i}{dX}$ and introducing it into eq. 14 yields

$$\frac{df_i}{dX} = \frac{\partial f_i}{\partial X} + \frac{\partial f_i}{\partial u_i} \cdot (-\tilde{K}_i^{-1}) \cdot \left( \frac{d\tilde{K}_i}{dX} u_i - \frac{dF_i}{dX} \right).$$ \hspace{2cm} (16)

Here, we introduce the adjoint state variable $\Lambda$, that is evaluated from

$$\Lambda_i^\top = \frac{\partial f_i}{\partial u_i} \cdot (-\tilde{K}_i^{-1}) \text{ or } \tilde{K}_i \Lambda_i = -F_i,$$ \hspace{2cm} (17)

respectively. Hence, using eq. 11 we notice $\Lambda = -u$ and finally find, after little rearrangement,

$$\frac{df_i}{dX} = 2u_i^\top \frac{\partial F_i}{\partial X} - u_i^\top \frac{d\tilde{K}_i}{dX} u_i.$$ \hspace{2cm} (18)

Thus, we need to evaluate $\frac{\partial F_i}{\partial X}$ and $\frac{d\tilde{K}_i}{dX}$ to calculate the desired derivative. Both $F_i$ and $\tilde{K}_i$ depend on the considered angular frequency $\lambda_i$, i.e. we require $\frac{d\lambda_i}{dX}$. A derivation of the following formula may be found in Fox and Kapoor [1968]:

$$\frac{d\lambda_i}{dX} = \frac{1}{2\lambda_i} \phi_i^\top \left( -\lambda_i^2 \frac{dM}{dX} + \frac{dK}{dX} \right) \phi_i.$$ \hspace{2cm} (19)
Result: \[ f_i, \frac{df_i}{dX} \] for all \( i \in \{1, ..., i_{\text{max}}\} \)

1. \( \lambda_i, \Phi_i \leftarrow \text{solve} \left( -\lambda_i^2 M + K \right) \phi_i = 0 \) for at least \( i_{\text{max}} \) modes; \hspace{1cm} // Eq. 1
2. \( \Phi \leftarrow \text{determine} \); \hspace{1cm} // Eq. 4
3. \( \frac{dM}{dX}, \frac{dD}{dX} \) and \( \frac{dK}{dX} \leftarrow \text{calculate using complex step method} \)
4. \( \text{for } i \leftarrow 1 \text{ to } i_{\text{max}} \text{ do} \)
5. \( \frac{d\lambda_i}{dX} \leftarrow \frac{1}{\lambda_i^2} \Phi_i^\top \left( -\lambda_i^2 \frac{dM}{dX} + \frac{dK}{dX} \right) \phi_i \); \hspace{1cm} // Eq. 19
6. \( u_i \leftarrow \left( \left( \Phi^\top K_i \Phi \right)^{-1} \cdot \left( \Phi^\top F_i \right) \right)^\top \Phi \); \hspace{1cm} // Eq. 5
7. \( f_i \leftarrow |F_i^\top u_i| \); \hspace{1cm} // Eq. 12
8. \( \frac{df_i}{dX} \leftarrow \text{assemble and calculate} \); \hspace{1cm} // Eq. 18
9. \( \text{end} \)

Figure 4: Algorithm for dynamic compliance and its sensitivity

Summarizing, the calculation of \( \frac{df_i}{dX} \) essentially rests on the calculation of \( \frac{dM}{dX}, \frac{dD}{dX} \) and \( \frac{dK}{dX} \). All these terms are calculated by means of the complex step method, which, in contrast to the finite differences approach, is not subject to subtractive round-off errors with decreasing step size. See Squire and Trapp [1998] or Jin et al. [2010] for more details. Hence, these derivatives are as accurate as machine precision permits.

The entire procedure is summarized in the algorithm given in fig. 4. The most costly part of the process is the determination of the angular frequencies and eigenvectors, given in line 1, and the calculation of \( \Phi \), line 2. As the state variable \( u_i \) is approximated as a superposition of few vectors and the adjoint state \( \Lambda_i \) is proportional to \( u_i \), we avoid the need to solve a great number of linear systems.

**Mesh update**

Having determined the mesh sensitivities \( \frac{df_i}{dX} \) for all \( i \), we may now step forward to update the finite element mesh accordingly in order to facilitate a shape optimization. As the sensitivities stemming from an adjoint calculation tend to be noisy and we want to retain smooth surfaces throughout the optimization, it proves beneficial to increase the regularity of the sensitivity field before it is scaled and applied as shape update.

Bletzinger [2014] introduced a method called Vertex Morphing that suits this task. The method requires the user to define a filter width, that determines the corresponding influence area and avoids design details, that are smaller. In order to apply Vertex Morphing, a sparse filter matrix needs to be constructed, that is then multiplied twice with the mesh sensitivity field. Hence, the computational cost is low. For an application of Vertex Morphing in the context of a freeform optimization of a flow duct, see Lachenmaier et al. [2020].

Vertex Morphing is used to determine and prescribe the translation of the nodes on the design surface depicted in blue in fig. 1. The translation of inner nodes is achieved by applying an established mesh motion technique, Shontz and Vavasis [2010]: An additional linear system of equations is solved for this purpose, that is based on an adapted version of the stiffness matrix \( K \). The adaptation being that the local stiffness matrix \( K_e \) of each element \( e \) is divided by its volume before assembly into a global matrix. This stiffens small elements and thus helps retaining a high mesh quality throughout the morphing as the movement rests on large elements.
The right hand side of the linear system contains both the prescribed translations of surface nodes according to the Vertex Morphing results and fixes the nodes on non-design surfaces. Having solved this linear system and scaled its result appropriately, we may translate each corner node accordingly. The edge nodes of the quadratic tetrahedral elements are translated such that they remain right in the middle of their respective corner nodes.

This step is repeated at the end of each iteration.

OPTIMIZATION AND RESULTS

The 12th eigenfrequency of the blade is high enough such that it cannot be excited by the guide vane induced engine order, as this resonance lies outside the operational range of the studied radial turbine. Hence, we consider \( i_{\text{max}} = 11 \) resonances for ND0. In order to conduct an optimization, we thus need to aggregate the \( \bar{f}_i \) for \( i = 1, \ldots, 11 \) into a single cost function. We use the \( p \)-norm with \( p = 5 \) for this purpose

\[
 f_{\text{sum}} = \left( \frac{1}{i_{\text{max}}} \sum_{i=1}^{i_{\text{max}}} \bar{f}_i^p \right)^{\frac{1}{p}}. \tag{20}
\]

The steepest descent algorithm in combination with the mesh updating strategy described above is applied to steer the optimization progress. The step size is constant and the optimization runs for 15 iterations. The respective convergence plot is given in fig. 5. In addition to \( f_{\text{sum}} \), we also plot \( \max_i \bar{f}_i \) for every iteration to illustrate that the continuously differentiable \( p \)-norm is well-suited to mimic the \( \max \)-norm. Clearly, the optimization is a success as we reduce \( f_{\text{sum}} \) by approximately 40%.

In fig. 6, we compare the baseline design (D1) and the optimized design (D15). In blue, we see the dynamic compliance \( \bar{f}_i \) for all 11 modes and both designs. We find that \( \bar{f}_{11} \) has the highest response in D1. The optimizer successfully reduces that value until finally \( \bar{f}_6 \) is the highest value for D15. In green, the figure also visualizes the maximal Von Mises stresses \( \max \sigma_{\text{vM}} \) within the volume for both D1 and D15 and each resonance. Clearly, the correlation between stress and dynamic compliance is very high. Lastly, we visualize the modal damping

Figure 5: Convergence plot, normalized with baseline design

Figure 6: Dynamic compliance, maximal stress and modal damping of D1 and D15

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ζi in red. We find that the shape change hardly affected the applied damping. We also noted that there is only a very slight shift in the eigenfrequencies.

In order to understand how the shape optimizer managed to reduce $\bar{f}_{11}$, we need to study the shape of the 11th eigenmode for both D1 and D15, see fig. 7. We find that D1 has a strong anti-node at the leading edge that matches the position of attack of the applied excitation vector $\mathbf{F}_{\text{const}}$. In D15, this anti-node is much less pronounced and thus explains the reduction of $\bar{f}_{11}$.

To study how D1 and D15 actually differ on the design surface, a difference plot is given in fig. 7 for both suction and pressure side. Herein, red surfaces represent areas where material has been added and blue areas show where the optimizer reduced material. As the turbine’s inlet diameter ranges around 230mm, the extent of the shape changes appears fairly small. This is positive in the sense that such small changes to the geometry seem feasible with regard to its affect on other, typically considered aspects, such as aerodynamics, polar moment of inertia or stress. Yet, it raises the question, whether the proposed shape changes lie in the range of the manufacturing tolerances of the casting process.

![Figure 7: Deflection of the 11th eigenvector for D1 (left) and D15 (right)](image1)

Based on the CFD setup presented and validated in Lachenmaier [2023], we compare one speed line of D1 and D15 in terms of both efficiency and mass flow. The studied speed line is $(n_{rps}/T_{t,in}) \cdot D_{in} = 307 \text{ m\cdotrpm} / \sqrt{K}$, where $D_{in}$ is the turbine inlet diameter. In fig. 9, both the mass flow parameter

$$\dot{m}_{\text{red}} = \dot{m} \cdot \sqrt{T_{t,in}/p_{t,in}}$$

(21)
and the turbine efficiency

\[ \eta_{ts} = \frac{T_{t,in} - T_{t,out}}{T_{t,in} \cdot \left(1 - \frac{1}{\Pi_{ts}}\right)} \quad (22) \]

are normalized w.r.t. to maximal values of D1. We find that the differences between D1 and D15 in terms of aerodynamic performance are negligible.

**CONCLUSIONS**

A forced response freeform optimization of a radial turbine has been conducted successfully. For this purpose, a simulation and optimization setup has been built that comes at little computational cost: This is a consequence of the primal LSE being solved by means of a superposition of few vectors and that a cost function (the dynamic compliance) has been chosen, that allows for a particularly cheap adjoint evaluation. Before the shape is updated in every iteration, Vertex Morphing is used to increase the regularity of the calculated shape gradient.

The model is built to incorporate an aero-acoustically motivated damping model. Additionally, the applied load is non-constant and respects that the aerodynamic load and excitation typically grow with rotation rate. The FEM solver used to solve this problem is a custom implementation written in Scilab.

The optimizer takes few iterations to reduce the studied cost function drastically. As it turns out, mode 11 has the highest response in this study. The reduction in this mode’s response can be traced to a change of the shape of the eigenvector, i.e. a reduction of the extent of the anti-node at the leading edge. The resulting shape changes prove small, which raises the question whether they range within manufacturing tolerances. The effect on the aerodynamic performance appears negligible.

The implementation of the optimization method proves simple, given that some basic adjoint capabilities, such as eigenfrequency sensitivities, are already available. Hence, incorporating it into existing, possibly multidisciplinary gradient-based radial turbine optimization workflows is feasible with affordable effort.
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