ANALYSIS OF THE THERMO-ACOUSTIC RESPONSE OF AN AERO-ENGINE INJECTOR USING COMPRESSIBLE LES

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ABSTRACT

Lean-burn combustion technologies are becoming increasingly attractive for the next generation of aero-engines due to their emission reduction potential. However, this concept comes with the downside that the flame becomes more susceptible to thermo-acoustic instabilities. To enable the implementation of such a concept, predictive simulation methods are desirable. In this work, a reactive compressible large-eddy simulation framework was applied to a full-scale lean-burn aero-engine injector under realistic operating conditions. The thermo-acoustic response of the injector was measured in the Rolls-Royce SCARLET rig using optical and acoustic measurement technologies. Advanced postprocessing techniques were employed to obtain optically measured flame transfer function (FTF) based on OH* chemiluminescence and the acoustic response characterized by measured transmission coefficients. These data sets enabled the assessment of the high-fidelity modeling framework over several frequencies for an application-oriented configuration.

KEYWORDS

THERMO-ACOUSTICS, ACOUSTICS, COMBUSTION, AERO-ENGINE, INJECTOR, LARGE EDDY SIMULATION

INTRODUCTION

Understanding and predicting thermo-acoustic instabilities has been of growing importance in the area of the developing and designing of new aero gas turbines. For the global goal of lower pollutant emissions in aviation, next-generation engines increasingly rely on lean combustion systems (Lefebvre and Ballal, 2010). Here, mixing air and atomized liquid fuel before combustion leads to the formation of a flame with lean-premixed characteristics, which can significantly reduce NOₓ and particulate emissions. However, the flame formation is more sensitive against the mixing process, and the engine is more prone to thermo-acoustic instabilities.

The acoustic response of flames to upstream perturbations is usually characterized using the concepts of the flame transfer function (FTF) and flame transfer matrix (FTM) (Paschereit et al., 1999; Schuermans et al., 2004). They can be employed in acoustic network models representing the whole combustor to assess the stability properties of the system. However, the procedure to obtain the acoustic flame response is the challenging and critical part of such an analysis.

In principle, both experimental and numerical approaches are available, which are well-validated and demonstrated in the literature using laboratory-scale model burners (see e.g. Tay-Wo-Chong et al. (2011); Kuhlmann et al. (2022); Merk et al. (2019); Joo et al. (2021)). While these studies are invaluable in terms of establishing the state-of-art and demonstration of model
capabilities, there exists a large gap between the idealized configurations and an actual engine environment in terms of complexity. Experimental setups targeting the measurement of the acoustic response of industrial gas turbine burners are rare and have only recently started to appear in the literature (e.g., Alanyalioglu et al. (2022), Treleaven et al. (2021), Venkatesan et al. (2022) and Apeloig et al. (2015)). The Rolls-Royce SCARLET (SCaled Acoustic Rig for Low Emission Technologies) rig (Fischer and Lahiri, 2021; Fischer et al., 2018), detailed in the subsequent section, was designed specifically to obtain the acoustic response of flames produced by full-size aero-engine injectors under single-sector equivalent engine operating conditions. It provides the possibility to measure the flame response in terms of acoustic data sets, using a multi-microphone method (Paschereit et al., 1999), and optical measurements capturing OH* chemiluminescence (CL) as an indicator for the heat release rate. Besides realistic operating conditions, the geometry and cooling technology of the combustor closely follows the modern annular gas-turbine combustor. Due to its wide range of operating conditions and overall design being of direct practical relevance, the data obtained is unique in the sense of developing and validating numerical approaches. To the authors’ knowledge, a combined experimental and numerical analysis of a similar setup over a range of frequencies has not yet appeared in the literature.

Therefore, the objective of this work is to use a high-fidelity reactive multiphase compressible large eddy simulation (LES) approach to numerically reproduce the flame response as measured in the SCARLET rig by means of the optical flame transfer function and acoustic transmission characteristics. In that sense, this work aims to contribute to bridging the gap between academia and industry by applying simulation methods validated on model configurations in an engine-like environment.

EXPERIMENTAL CONFIGURATION

The SCARLET rig (Fischer and Lahiri, 2021; Fischer et al., 2018) is a specialized setup to perform testing of full-scale aero-engine injectors under realistic single-sector operating conditions within an acoustically controlled environment. A schematic overview is given in Figure 1. The rig is formed by a test section placed between two constant diameter ducts and is fed by a supply of high-pressure pre-heated air through the entrance section of the upstream duct. The overall mass flow rate through the rig is controlled by an adjustable area choked nozzle located after the downstream duct. Different injector pressure drop conditions can be tested through manipulation of the upstream total pressure and nozzle throat area. Each duct features a pair of sirens to provide monotone acoustic excitation, a set of five flush mounted microphones, and a damper system to reduce the acoustic reflections at the end sections to avoid excitation of longitudinal acoustic modes of the setup. The test section, shown in Figure 2, consists of the combustion chamber and an outer annulus that are separated by a conically shaped perforated liner that provides effusion cooling. The liner also contains an interface for the endoscope which provides optical access for the OH* chemiluminescence imaging system. The bulk mass flow of \( \approx 2 \text{ kg s}^{-1} \) within the upstream duct separates into three streams within the test section, shown
as arrows in Figure 2. Around 60% is fed through the injector, and the remaining air is split between the perforated liner (red arrows) and the heat shield cooling passages (blue arrows), both feeding into the combustion chamber. The investigated operating point is characterized by

\[ T_{\text{in}} \approx 700 \text{ K}, \quad p_{\text{in}} \approx 12 \text{ bar} \]

and a global equivalence ratio of \( \phi_{\text{glob}} = 0.34 \), corresponding to a thermal power of approximately 2 MW. The design of the investigated lean-burn injector consists of three individual passages: pilot, middle and outer passages. Each passage contains axially positioned vanes to induce swirl in the flow. Liquid Jet-A1 fuel is injected from two different locations, using a pressure atomizer at the pilot passage and a prefilming airblast atomizer located in the intersection of the middle and outer passages. The fraction of the total fuel flow rate fed into these individual injection systems is adjustable. For the operating point under investigation, \( \approx 20\% \) of fuel flow was injected through the pilot injector nozzle.

The experimental campaign consisted of a set of five experiments targeting five distinct frequencies. Based on the operating conditions and diameters of upstream and downstream ducts, acoustic modes at frequencies of interest are well below the cut-off frequency and hence they propagate as plane waves. The CL imaging system consists of an intensified relay optics (IRO) device, and a high-speed CMOS camera. For all measurements, the IRO gain was set to 70% with an IRO gate window of 90 \( \mu s \) and camera exposure time of 200 \( \mu s \), providing 21000 images sampled at a rate of 5 kHz. The microphone data and camera trigger were synchronized in time using a reference signal recorded simultaneously by both the microphone data acquisition system (DAQ) and the DAQ used to control the camera. The applicability of \( \text{OH}^* \) CL based FTF measurement relies on the assumption that the global heat release rate in the combustor scales linearly with the measured global (integrated) \( \text{OH}^* \) intensity. This was verified by inspecting the relation between global \( \text{OH}^* \) intensity and thermal power at different operating conditions.

**NUMERICAL SETUP**

In this section, an overview of the employed numerical tools, physical models, computational grid and the boundary conditions are provided.

**Numerical Tools and Models**

For the solution of Navier-Stokes equations, the Rolls-Royce in-house CFD code PRECISE-UNS (Anand et al., 2013) was employed. It is a parallel, finite-volume based code for arbitrary unstructured grid systems and employs a pressure correction method with extension to compressible flows (Klapdor et al., 2013) as the solution algorithm. Large eddy simulations (LES) follow the implicit filtering approach, where the computational grid acts as the spatial filter op-
erator. The subgrid viscosity was computed using the σ model using a global model constant of $C_\sigma = 1.35$ (Nicoud et al., 2011). For the present LES, all diffusive fluxes and the pressure correction equation were spatially discretized using a second-order central scheme. The convective fluxes were discretized using a second-order total variation diminishing (TVD) scheme (Jasak et al., 1999). Unsteady terms were discretized using second-order backward differences. The spray evolution during liquid fuel injection is described by a Lagrangian particle tracking method, taking into account the two-way interactions between the gas and liquid phases.

The combustion process is modeled using the Flamelet Generated Manifold (FGM) approach developed by Oijen and Goey (2000). This method relies on a tabulated set of pre-computed thermodynamic states by means of one-dimensional flamelet simulations using detailed chemistry, which are then parameterized by several control variables. Instead of solving for individual species transport equations, the scalar transport equations for FGM control variables are solved simultaneously with the governing equations for fluid flow. The local thermodynamic state and required chemical source terms are retrieved from the FGM table. The set of one-dimensional flamelet computations is transformed from physical space to composition space based on the mixture fraction $Z$ and progress variable $Y$. The progress variable is selected as the sum of specific mole fractions of $H_2O$, $CO_2$ and $H_2$. $C_{12}H_{26}$ is used as an surrogate fuel for the Jet-A1 fuel, whose chemical kinetics are modeled based on the detailed mechanism of Nehse et al. (1996). For the generation of flamelet solutions, the laminar one-dimensional flame solver CHEM1D (Hermanns, 2001) is employed. A set of freely propagating flamelets are computed within the flammability limits. The unburnt conditions are specified as $p = p_{out}$, $T_{air} = T_{in}$, and $T_{fuel} = 600$ K where the fuel temperature is a representative value for the vaporization temperature of Jet-A1. In the context of LES, only filtered scalars are transported, which cannot be used directly for the table access. The unresolved subgrid distribution of the transported scalars is modeled by a presumed probability density function (PPDF) approach. In this work, for the mixture fraction a $\beta$-PDF and for the progress variable a three-$\delta$ PDF was assumed (Eggels, 2018). The final FGM table is parameterized by the four control variables $[\tilde{Z}, \tilde{Y}, \tilde{Z}^{n2}, \tilde{Y}^{n2}]$, where $\langle \cdot \rangle$ and $\langle \cdot \rangle^{n2}$ denote Favre averaging and variance respectively. The corresponding transport equation for each control variable is solved simultaneously with the governing equations for the fluid flow, using a value of 0.6 for the turbulent Schmidt and Prandtl numbers.

Within the assumptions of the FGM approach, the local thermodynamic state is completely determined by the local values of the control variables. For compressible flow, the local value of static enthalpy is influenced due to compressibility effects, which causes the local thermodynamic state to deviate from the one described by the FGM table. Since the Mach number of the flow is low overall compressibility effects are negligible. Therefore, the local chemical composition is not altered and the static temperature was computed using a linear expansion of temperature around the tabulated state (Vicquelin et al., 2011). Using the combustion model described in this section within the context of thermo-acoustics, we assume that the contributions of pressure fluctuations and unsteady effects that could alter the description of the flame structure are negligible on the observed heat release rate fluctuations.

**Computational Domain**

The solution domain is spatially discretized using a structured grid of 11.5 million hexahedral elements to ensure good orthogonality and preserve accuracy during the computation of fluxes. In order to generate a structured grid, small geometrical details are excluded from the
geometry. The injector geometry exactly follows the CAD model. The influence of these simplifications on the non-reacting acoustic scattering behavior was previously studied in (Alanyalioglu et al., 2022) and was shown to be negligible. The distribution of a characteristic element size defined as $\Delta = \sqrt[3]{\Delta V}$ ($\Delta V$ denotes cell volume) and an overview of the computational grid within the combustor is shown in Figure 3. The cells within the injector passages have the identical size to cells in the region immediately downstream of the injector. The quality of the LES was estimated using the $IQ_\nu$ criterion introduced by Celik et al. (2005), assuming that the numerical viscosity is proportional to the local mean sub-grid viscosity with a factor of 50%. Within the combustor, this yields an $IQ_\nu$ distribution of up to $\approx 85\%$ at the regions of the recirculation zones filled with burnt gas, and lowest values of $\approx 70\%$ at high-velocity regions of the jet originating from the outer injector passage.

Away from the combustor, inside the upstream and downstream duct, the grid was progressively stretched to reduce the overall element count. While doing so, it was ensured that wavelengths of the excitation frequencies of interest were resolved using at least 50 cells along the propagation direction within the ducts.

**Boundary Conditions**

In this section, relevant details concerning the numerical modelling of the boundary conditions are described. All other boundaries that are not referred to in the remainder of this section are treated as adiabatic no-slip walls.

**Effusion Cooling**

Physically resolving the effusion cooling holes in the computational domain of SCARLET requires a significant amount of cells, rendering an LES computationally unfeasible. Therefore, regions of effusion cooling are modeled using spatially uniform effusion boundary conditions, which implement the necessary source terms to model the injection/suction of fluid. In order to use these boundary conditions, the internal flow split fractions through the associated surfaces need to be known in advance. The values of the flow split fractions were obtained as described in Alanyalioglu et al. (2022). It is worth noting here that the acoustic behaviour of these boundary conditions is identical to that of a rigid wall, i.e., $u' = 0$.

**Inlet and Outlet Boundaries**

The inlet and outlet boundaries are placed at the upstream and downstream siren locations, respectively. At these boundaries, Navier-Stokes characteristics-based boundary conditions (Poinset and Lele, 1992) with plane wave masking (PWM) (Polifke et al., 2006) are employed. This ensures almost non-reflecting acoustic behavior when subjected to plane waves. Acoustic
excitation from the inlet is conducted via the specification of incoming acoustic wave amplitude variation using a prescribed acoustic forcing signal. As the experimental measurements target five distinct frequencies, a broadband signal using a combination of cosine waves at these frequencies is employed for the simulation. To limit the instantaneous amplitude of the signal, the individual cosine waves are combined using Newman’s phase angles (Newman, 1965), which yield a low crest factor of 4.2 dB for five tones. This forcing signal yields a maximum peak-to-peak $p'/p_{in}$ value of approximately 1%, in which the acoustic wave propagation is assumed to be linear Davies (1988). This was later verified through inspection of area-averaged $p'$ and $u'$ obtained at the sampling planes in the duct sections. It is known that swirl stabilized flames may exhibit a non-linear response especially when subjected to high excitation amplitudes [REF]. This is associated with the complex convective dynamics of the combustor rather than the linearity or non-linearity of acoustic wave propagation. Due to signal-to-noise ratio limitations, an experimental study to verify the measured response shows a linear behaviour was not possible. To minimize the associated uncertainty, present LES was conducted using the same excitation amplitudes for each tone as in the experiments.

Liquid fuel injection

Lagrangian parcels representing a set of Jet-A1 fuel droplets with identical initial velocity, diameter and temperature were injected at prescribed ring-shaped discrete regions adjacent to the pilot injector nozzle and edge of prefilming airblast atomizer. At each time step, the diameters of each injected parcel were sampled from a Rosin-Rammler distribution with prescribed statistical properties. In addition to that, the initial values of velocity components of the injected parcels were randomly varied within a prescribed fraction of their mean values. The resultant droplet distribution is assumed to correspond to the post-atomization products of the atomizers. As no experimental data is available regarding the droplet characteristics, the parameters referred in this section were prescribed according to the Rolls-Royce Deutschland experience from similar configurations. At wall boundaries, Lagrangian parcels were assumed to reflect obeying the conservation of momentum, and no sticking was allowed. The evaporation rate of the droplets was computed using the model developed by Chin and Lefebvre (1983).

DATA PROCESSING

As the quantities of interest are not directly available in the raw data obtained from both experimental and numerical approaches, a post-processing step needs to be carried out, which is detailed in this section. The primary quantity of interest is the FTF, defined as

$$ \text{FTF}(\omega) = \frac{\overline{\dot{Q}'(\omega)}}{\overline{u'_{\text{ref}}(\omega)}} $$

where $\omega = 2\pi f$ is the angular frequency, $\overline{\dot{Q}'} = \dot{Q}'/\overline{\dot{Q}}$ denotes the normalized global heat release rate fluctuations and $\overline{u'_{\text{ref}}} = u'/\overline{u_{\text{ref}}}$ denotes normalized velocity fluctuations at a reference location. Since the primary measurement outcome of the SCARLET rig is the acoustic states in the upstream and downstream ducts, the coefficients of the acoustic scattering matrix are also of interest. Combining these two quantities allows for an in-depth analysis of optical and acoustic data to provide additional insights. The process of experimental and numerical computation of the scattering matrix within the context of the SCARLET rig was previously detailed in (Alanyalioglu et al., 2022), and for brevity, it is not repeated here.
**Experimental Data**

For experiments, the instantaneous value of the global OH* signal is assumed to be proportional to the instantaneous global heat release rate. By this, the FTF can be calculated from the normalized fluctuations of the global CL intensity I, which is defined as the summation of individual pixel intensities divided by the pixel count for each CL image. The normalized fluctuations of the global intensity \( I' \) are then obtained by subtracting the mean image from the instantaneous one and normalizing it by the mean image.

The conventional reference position for the FTF is the ‘cold’ state immediately upstream of the flame, such that (within the context of plane waves) the incoming acoustic wave is only altered due to the flame after the reference position. However, this is not possible for the present case since experimental velocity information is only accessible within the duct regions. The acoustic velocity fluctuations are obtained indirectly from the plane wave decomposition (PWD) using the multi-microphone method, which can be used to compute the acoustic velocity at any location within the duct. The plane joining the upstream duct and the test section was selected to establish a common reference position \( x_{\text{ref}} \) for the experiment and the simulation, and \( \bar{u} \) is evaluated at \( x_{\text{ref}} \) using the mean velocity in the upstream duct as \( \bar{u}_{\text{ref}} \). The FTF is then computed as

\[
\text{FTF}_{\text{CL}}(\omega) = \frac{P_{u'I'}(\omega)}{P_{u'u'}(\omega)}
\]  

where \( P_{xy} \) denotes the one-sided cross power spectrum of the signals \( x(t) \) and \( y(t) \), and \( P_{xx} \) denotes the one-sided power spectrum of \( x(t) \). All power and cross-spectrum computations required while post-processing the experimental data were conducted using Welch’s method with 50% overlap, Hann window function, and a segment length of 1 s to provide a frequency resolution of 1 Hz. Following the selection of the reference plane, it is technically more precise to refer to the FTF under consideration as an injector flame transfer function since the reference location is based on the acoustic state upstream of the injector.

**LES Data**

The global heat release rate is directly available from simulations by integrating the local heat release rate over the simulation domain \( \dot{Q} = \int \dot{q}(x) \text{d}V \). A set of local PWD’s were performed on upstream duct sampling planes to obtain the velocity fluctuations at the reference location. It uses area-averaged values of \( p' \) and \( u' \) obtained from sampling planes corresponding to the experimental microphone locations. The downstream and upstream travelling plane wave components \( f \) and \( g \) are obtained and are then magnitude-averaged in frequency domain to yield a best fit applicable throughout the duct. Finally, \( u' \) at \( x_{\text{ref}} \) was computed by introducing the corresponding phase difference to the phase angles of \( f \) and \( g \) at the nearest sampling plane. Having obtained \( \bar{u}' \) at the reference plane, the FTF was then computed using

\[
\text{FTF}_{\text{LES}}(\omega) = \frac{P_{w'Q'}(\omega)}{P_{w'u'}(\omega)}
\]  

Since the simulation data has a lower sample count compared to the experiments, no spectrum averaging was conducted as it further reduces the frequency resolution.
RESULTS

An initial LES with standard mass flow inlet and pressure outlet boundary conditions was carried out for a physical duration of approximately 75 ms until statistical convergence was achieved. This mean result was used to set the reference acoustic state to allow proper wave identification for PWM. Subsequently, a simulation with upstream acoustic excitation was conducted. The acoustically excited LES was run for a duration of 110 ms, where the first 10 ms were considered as the initial transient and not used in the subsequent analysis. The remaining 100 ms provide a resolution of 10 Hz in the frequency domain. For all LES, a time step of 1 µs was used for accurate resolution of convective dynamics, which provides a convective CFL number of less than unity. Using this time step, the period corresponding to the highest frequency of interest is resolved with more than 1000 time steps. The acoustic CFL number is also less than unity, except in the regions of small cells near the injector where the highest value is \( \approx 6 \). The results in the frequency domain are presented in terms of normalized frequency \( St = f \tau_c \) where \( \tau_c \) is a representative time scale of the injector. This value is computed as \( \tau_c = D/U_{\text{ref}} \), where \( D \) is the diameter of the injector and \( U_{\text{ref}} = \sqrt{2(p_{\text{in}} - p_{\text{out}})/\rho_{\text{in}}} \).

Main Characteristics of the SCARLET Rig

Before an in-depth analysis of the excited LES is provided, the main characteristics of the combustor region are discussed using mean and instantaneous contour plots. These are depicted in Figure 4 for the axial velocity, temperature, equivalence ratio, and volumetric heat release rate. The axial velocity indicates the main features of the flow field inside the combustor. The highly turbulent swirled main jet forces the flow towards the effusion cooling liner, resulting in a large recirculation zone. In the center of the domain, the recirculation interacts with the counter-rotating pilot jet. Towards the downstream duct, a rapid flow acceleration is evident, caused by the thermal expansion and reduction of the cross-section. Small turbulent structures can be identified in the shear layer of the recirculation zone close to the injector. However, further downstream, only larger structures are present.

Inspecting the contours of temperature, distinct regions of the pilot flame and the main flame can be identified. The pilot flame is stabilized inside the shear layer between the counter-rotating main and pilot jets. The turbulence in this region interacts with the reaction, as indicated by the strongly wrinkled flame. The main flame is stabilized by the hot flue gas from the pilot flame and recirculation zone. In regions where the main flame interacts with the liner, mixing the effusion air with the flue gases reduces the local temperatures significantly. The effect of the effusion cooling air is evident along the entire liner and also visible in the downstream duct.

The mixing process, driven by the unsteady nature of the turbulent flow, happens in a very...
short length scale as the instantaneous contours of equivalence ratio suggest. To indicate the spray penetration, the equivalence ratio is overlaid with black in regions of liquid mixture fraction greater than 0.05. Pockets of the rich mixture can be carried to the flame front and strongly influence the local reactions. By design, the pilot and main flames have different characteristics. The pilot flame has more non-premixed-like flame characteristics, having an equivalence ratio close to unity where most pilot fuel is consumed. On the contrary, due to the mixing processes between the zone of injection and combustion, the main flame is lean and premixed, having a mean equivalence ratio of $\approx 0.5$.

These two distinct flames are also visible in the volumetric heat release. Although the peak intensity of volumetric heat release does not occur in the main flame, it contributes more than 75% of the total heat release since most of the liquid fuel is injected from the main atomizer. Again, the strongly wrinkled flame front and significant local variations of the heat release value demonstrate the substantial influence of turbulent mixing of the reaction inside aero-engine combustors. This is a possible mechanism for combustion instabilities, as the arrival times of such structures to the flame front can be influenced by acoustic velocity fluctuations and could be one of the reasons to explain why the lean-burn injectors tend to be more sensitive compared to Rich-Quench-Lean (RQL) injectors.

**Flame transfer function**

After this brief introduction to the overall characteristics of the configuration, the FTF obtained from LES based on global heat release rate is shown along with the experimental FTF based on OH* CL in Figure 5. Besides small discrepancies in the gain for the lowest and the phase of the highest tone, both FTFs agree well in general. In particular, the phase angle in the simulation is excellently predicted, considering the complex configuration under investigation. The phase angle represents the time lag between the response ($\dot{Q}'$ or OH* signal) and the reference velocity, which is established due to convective dynamics. Hence, it is strongly connected with the underlying physics inside the system. Based on this good prediction of the phase slope, one can conclude that a compressible LES can capture the driving physics of real aero-engine injectors. At this point, it should also be noted that the CL image evaluation includes some uncertainties. In particular, the challenging alignment of the recording of the images with the microphone measurements in time results in uncertainties in the phase angle. Each image represents the cumulative OH* signal captured within the duration of the IRO gate window ($t_{IRO} = 90\,\mu s$). Hence, it represents an interval rather than an instant. For the evaluation of the CL based FTF, each CL image was assumed to correspond to the beginning of the IRO window. The phase uncertainty is estimated as $2\pi ft_{IRO}$ radians for a given $f$. This uncertainty
is within the range of square markers for experimental data in Figure 5. Spectrums of both $\mathcal{Q}'$ and $\mathcal{T}$ contain contributions due to the turbulent nature of the flame at frequencies of interest. Ideally, the flame transfer function should be constructed to remove these contributions using appropriate methods such that flame response due to excitation signal is isolated. However, it can be argued that the presented FTF is fully comparable if the background spectrum is present for both experiment and simulations. Since the employed modeling strategy has been proven to be of high fidelity, the background spectrum is considered to be captured adequately.

Due to the computationally expensive nature of LES, the length of practically realizable time series is much less when compared to the experimental measurements. This permits the application of a windowing and spectrum-averaging based technique to assess whether the computed response shows a convergent behavior on a tonal basis. The response obtained for the lower frequencies is expected to suffer more from an insufficient amount of samples, and should be noted as a possible contributing factor on the observed discrepancies.

Another possible source of deviation that is not accounted for in the present LES is the possible influence of acoustic excitation on the spray characteristics. In particular, the airblast atomization process assisted by the shear caused by can be affected by acoustic fluctuations (Christou et al., 2022; Ahn et al., 2018). Under such circumstances, the frequency response of the atomizer would be manifested as equivalence ratio fluctuations and have an influence on the overall spectrum of the flame response. In order to assess this further in the present LES framework, realistic atomization models that account for acoustic velocity fluctuations is required.

Scattering matrix

Another way of investigating the acoustic response of a flame is through analysis of its influence on plane wave components upstream and downstream of the flame. This is usually expressed as the acoustic scattering matrix, which is an experimentally measured quantity in the SCARLET rig. As it is a purely acoustic measurement, it provides an additional means of comparison besides the optically measured CL data. For upstream excitation, the upstream transmission coefficient $t^+$ is especially relevant since it contains the influence of the flame.

Using only an upstream excited simulation, it is possible to compute the upstream transmission coefficient $t^+$ and upstream reflection coefficient $r^+$, provided that the downstream end is acoustically non-reflective. Since the behavior of $r^+$ is not significantly influenced by the flame, only $t^+$ is considered here. In the following discussion, $f_u$ and $f_d$ denote the downstream traveling Riemann invariants in the upstream and downstream ducts, respectively. Similarly, $g_u$ and $g_d$ denote the upstream traveling Riemann invariants. Besides the lowest tone at $St \approx 0.05$, the downstream reflection coefficient $g_d/f_d$ was observed to be less than 0.045 in simulations, which we consider as non-reflective for the present analysis. This implies that $g_d \approx 0$ and $t^+ = f_d/f_u$. The reflection coefficient at the lowest tone was significantly higher, caused by inaccurate identification of $g_d$ using only the outlet plane, reducing the performance of PWM. The root cause of this issue is flow structures in the downstream duct with similar time scales, and proper identification of $g_d$ requires more advanced techniques such as in (Kopitz et al., 2005). Therefore, the lowest tone is omitted in the present analysis of $t^+$. The behavior of $t^+$ is given in Figure 6 in comparison with experimental measurement, where both upstream and downstream states were evaluated at $x_{ref}$. For $St \approx 0.35$, the upstream transmission coefficient is predicted accurately, while an over-prediction of acoustic transmission for lower frequencies followed by an under-prediction for the highest tone can be observed. The trend of the phase angle follows
Figure 6: Comparison of upstream transmission coefficient obtained from LES and experiment. Left: gain. Right: phase angle.

the experimental values after \( \text{St} \approx 0.26 \), with an almost constant offset. Overall, \( t^+ \) obtained from the LES contains stronger discrepancies to the experiments compared to FTF predictions in the previous section. Contrary to the FTF, the behavior of \( t^+ \) represents all acoustic phenomena that contribute to \( f_d \), which is not only due to the flame. The downstream acoustic state measured in the experiments is formed by portions of \( f_u \) which can follow two different acoustic paths: through the injector and the flame, or the outer annulus and the perforated liner as the secondary path. However, distinguishing these contributions in the experimental data is very challenging. In the LES, the perforated liner is modeled with previously mentioned boundary conditions, which can capture effusion cooling correctly, as discussed in the previous section. However, frequency-dependent transmission characteristics of the perforated liner are not considered in the simulation. Hence, there is only one acoustic path from the upstream duct to the downstream duct in the LES.

In the absence of the flame, it follows that the expected outcome from the simulation is a consistent under-prediction of \( t^+ \). This was observed in the analysis of the non-reacting acoustic scattering behavior of the SCARLET rig in (Alanyalioglu et al., 2022), which was conducted under similar operating conditions. The present work introduces the influence of the flame on \( f_d \), with further availability of the FTF using an independent measurement to aid the analysis. Taking into account that the FTF compares well between the present LES and experiment, it follows that the contribution of the flame on \( f_d \) should be reasonably captured as well. For frequencies less than \( \text{St} \approx 0.35 \), \( t^+ \) is over-predicted. Considering the good prediction for the FTF gain, this behavior is caused by damping and the phase relationship induced by the perforated liner. The inverse trends of the phase angle in this frequency range support this deduction, suggesting \( f_d \) is strongly influenced by the secondary path. At \( \text{St} \approx 0.35 \), both FTF and \( t^+ \) are predicted most accurate, which suggests that transmission through the secondary path is negligible at this frequency. This observation is in-line with Alanyalioglu et al. (2022), where the best prediction for the non-reacting \( t^+ \) was obtained for \( \text{St} \approx 0.37 \). This further strengthens the conclusion that the perforated liner has almost impermeable acoustic transmission characteristics near \( \text{St} \approx 0.35 \), such that the liner being acoustically impermeable in simulations becomes a good approximation. The liner and flame contributions are in phase for the highest tone at \( \text{St} \approx 0.45 \). Here, the flame has less influence on \( f_d \) since the gain of FTF is low, which leads to an under-prediction of the transmission coefficient. These observations suggest that a complete description of acoustics in the SCARLET rig needs to consider the perforated liner’s acoustic behavior.
CONCLUSIONS
This work presents the application of compressible LES for predicting the thermo-acoustic response of a full-scale lean-burn aero-engine injector. The good agreement of the FTF with experimental data over a wide range of frequencies demonstrates that the complex LES-based framework can capture the thermo-acoustic response of the flame inside an industrially relevant setup involving combined effects of turbulence, combustion and acoustics. Interestingly, the upstream reflection coefficient, also available in the experiments, exhibits more significant discrepancies, although the acoustic response of the flame is correctly captured. It is concluded that the acoustic behavior of the perforated liner, modeled acoustically impermeable in the LES, affects the transmission characteristics. Hence, future works could focus on improving the acoustic modeling of the perforated liner to capture the complete acoustic response in industrially relevant systems.

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